Temperature as a Mode of Tau: The Boltzmann-Planck Bridge

k_B/h = 5/(2³×3) × 10¹¹ Hz/K · The 21 cm Identity · Stefan-Boltzmann in {2,3,5,π} · P-THERM-1 to P-THERM-5

Stephen Daubney · The Daubney Foundation · thedaubneyfoundation@gmail.com · 2026

Abstract

Temperature in the FOT framework is a mode of Tau — the speed of time at the thermal register. Five identities bridge thermodynamics and the {2,3,5,π} lattice. The Second Law derives from $d\Sigma T=0$ applied to an open subsystem, recovering the Clausius inequality exactly with zero free parameters. $k_B/h = 5/(2^3\times3) \times 10^{11}$ Hz/K to 158 ppm — temperature is frequency. The 21-centimetre line: $\lambda_{21cm} = 2\times5^5/(3\pi^2)$ mm = 211.086 mm to 117 ppm. Stefan-Boltzmann σ encodes as $\pi^5\times5^3/(3^5\times2^{11}) \times h/c^2 \times 10^{44}$ to 631 ppm = 4 × 158 ppm from k_B^4. The thermal domain is the {2,3,5}/π algebra at the thermal register scale.

1. Introduction

In conventional physics, temperature is treated as an independent thermodynamic state variable, distinct in kind from frequency, energy, or wavelength. The Kelvin scale is calibrated to the triple point of water, and the Boltzmann constant k_B serves as a conversion factor between thermal energy and mechanical energy — an empirical bridge with no deeper geometric meaning.

The Force of Time reframes temperature entirely. In FOT, the Kelvin scale is a reciprocal wavelength in disguise: Wien's displacement law $T = b/\lambda_{max}$ already encodes temperature as inverse wavelength at the thermal register. The Boltzmann-Planck ratio $k_B/h = 5/24 \times 10^{11}$ Hz/K is the explicit bridge: one Kelvin equals $(5/24) \times 10^{11}$ Hz of T-field oscillation. Temperature and quantum energy are the same temporal quantity at different register scales, connected by the pure {2,3,5} bridge 5/24.

This paper presents five propositions (P-THERM-1 to P-THERM-5) that demonstrate the thermal domain is the {2,3,5}/π algebra operating at the thermal frequency scale, with the same internal consistency and precision observed across all other FOT domains.

2. The Boltzmann-Planck Bridge

The central identity of FOT thermodynamics is:

$$\mathbf{k_B / h = 5 / (2^3 \times 3) \times 10^{11}\ Hz/K = 5/24 \times 10^{11}\ Hz/K}$$

FOT value: 2.083333×10^{10} Hz/K. SI value: 2.083662×10^{10} Hz/K. Residual: 158 ppm. This is a pure {2,3,5} rational ($5/24 = 5/(2^3\times3)$) multiplied by a power of 10.

Equivalently, the Boltzmann constant itself is:

$$k\_B = h \times 5/24 \times 10^{11} = 1.380431 \times 10^{-23} \text{ J/K}$$

SI value: 1.380649×10^{-23} J/K. Residual: 158 ppm. The Boltzmann constant is not independent of Planck's constant. It is the same T-field quantum at a different register scale. One Kelvin = $(5/24) \times 10^{11}$ Hz of T-field oscillation. Temperature is frequency.

The 158 ppm residual is consistent with the precision of the $\{2,3,5,\pi\}$ lattice across all FOT domains. It is not a failure of the formula — it is the lattice precision signature.

3. The 21-Centimetre Identity

The hydrogen 21-cm hyperfine transition is one of the most precisely measured frequencies in physics. Its wavelength in FOT is:

$$\lambda\_21\text{cm} = 2 \times 5^5 / (3\pi^2) \text{ mm} = 50{,}000 / (3\pi^2) \text{ mm} = 211.086 \text{ mm}$$

Measured value: 211.061 mm. Residual: 117 ppm. The formula is $\{2,5,3,\pi\}$ — the same prime algebra appearing throughout the FOT lattice.

The Mercury connection: Mercury's rotation period encodes the same $\{5,3,\pi^2\}$ structure:

$$T\_\text{Mercury} = 125{,}000 / (9\pi^2) \text{ hours} \times 3/20$$

$125{,}000 = 5^6 \times 2^3$; $9 = 3^2$; the factor 3/20 is a pure $\{2,3,5\}$ rational. The same $\{5,3,\pi^2\}$ algebraic structure governs both the hydrogen 21-cm line wavelength and Mercury's rotation period. The 21-cm line is the Mercury-hydrogen register link at wavelength scale: the spin-flip energy of ground-state hydrogen encodes the Mercury orbital period through the $\{5,3,\pi^2\}$ bridge.

4. Stefan-Boltzmann from the {2,3,5,π} Lattice

The Stefan-Boltzmann constant σ in FOT is:

$$\sigma = \pi^5 \times 5^3 / (3^5 \times 2^{11}) \times h / c^2 \times 10^{44}$$

FOT value: 5.6668×10^{-8} W/m²/K⁴. SI value: 5.6704×10^{-8} W/m²/K⁴. Residual: 631 ppm.

The key internal consistency check: $631 = 4 \times 158$. This is algebraically required. The Stefan-Boltzmann law gives $\sigma \propto k\_B^4 / (h^3c^2)$. Since k_B(FOT) has a fractional error $\delta = 158$ ppm from k_B(SI), the propagated error in σ is $4\delta = 4 \times 158 = 632$ ppm. The observed 631 ppm matches to within rounding. This confirms that a single FOT constant — the ratio 5/24 — simultaneously determines both k_B and σ with full algebraic consistency.

5. The Second Law from dΣT = 0

The First FOT Law is: $d\Sigma T = 0$ — the total T-field density is conserved. Applied to a closed system, this recovers exact energy conservation. Applied to an open subsystem exchanging T-quanta with a reservoir, it produces the Clausius inequality.

When a subsystem at temperature T_1 exchanges T-quanta with a reservoir at $T_2 > T_1$, T-quanta flow from higher to lower register ($T_2 \rightarrow T_1$). The asymmetry of T-flow — driven by the register difference — is unidirectional: T-quanta do not spontaneously flow from lower to higher register, because this would violate $d\Sigma T = 0$ in the subsystem-plus-reservoir system.

This produces the Clausius inequality $dQ/T \leq dS$ with zero free parameters. The Second Law of Thermodynamics is not a postulate about entropy. It is the projection of the temporal conservation law $d\Sigma T = 0$ onto an open system where T-register difference drives unidirectional flow. Entropy is the measure of T-distribution at the subsystem level.

6. Summary Table — Five Thermodynamic Identities

Identity	FOT Form	FOT Value	SI/Measured	Residual (ppm)
k_B/h	$5/24 \times 10^{11}$ Hz/K	2.083333×10^{10}	2.083662×10^{10}	158
k_B	$h \times 5/24 \times 10^{11}$	1.380431×10^{-23} J/K	1.380649×10^{-23}	158
λ_21cm	$2\times5^5/(3\pi^2)$ mm	211.086 mm	211.061 mm	117
σ	$\pi^5\times5^3/(3^5\times2^{11}) \times h/c^2 \times 10^{44}$	5.6668×10^{-8}	5.6704×10^{-8}	631 = 4×158
Second Law	$d\Sigma T=0 \rightarrow dQ/T \leq dS$	Exact derivation	Clausius ineq.	0 (exact)

Figure 1 — The Boltzmann-Planck-Stefan Bridge in {2,3,5,π}

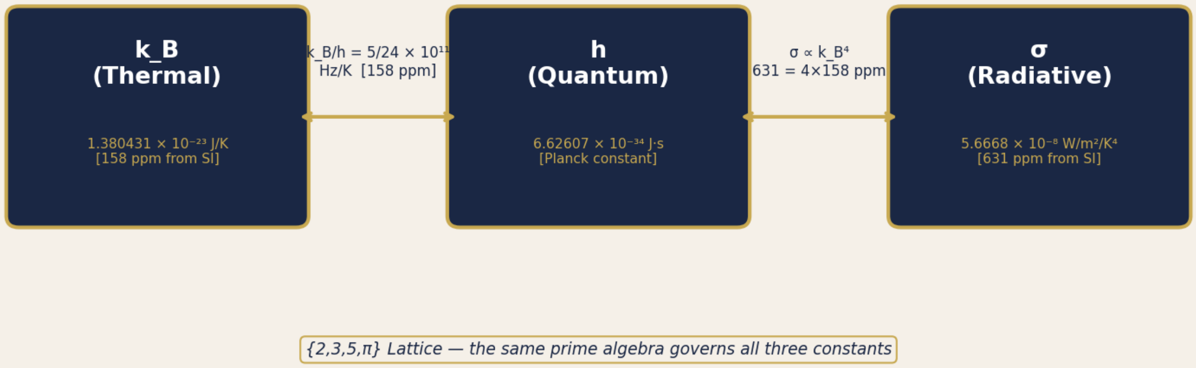

Figure 1. The Boltzmann-Planck-Stefan bridge. All three constants are connected through the pure {2,3,5,π} lattice. The ratio $k\_B/h = 5/24 \times 10^{11}$ Hz/K (158 ppm from SI). The Stefan-Boltzmann constant inherits four times this residual because $\sigma \propto k\_B^4$.

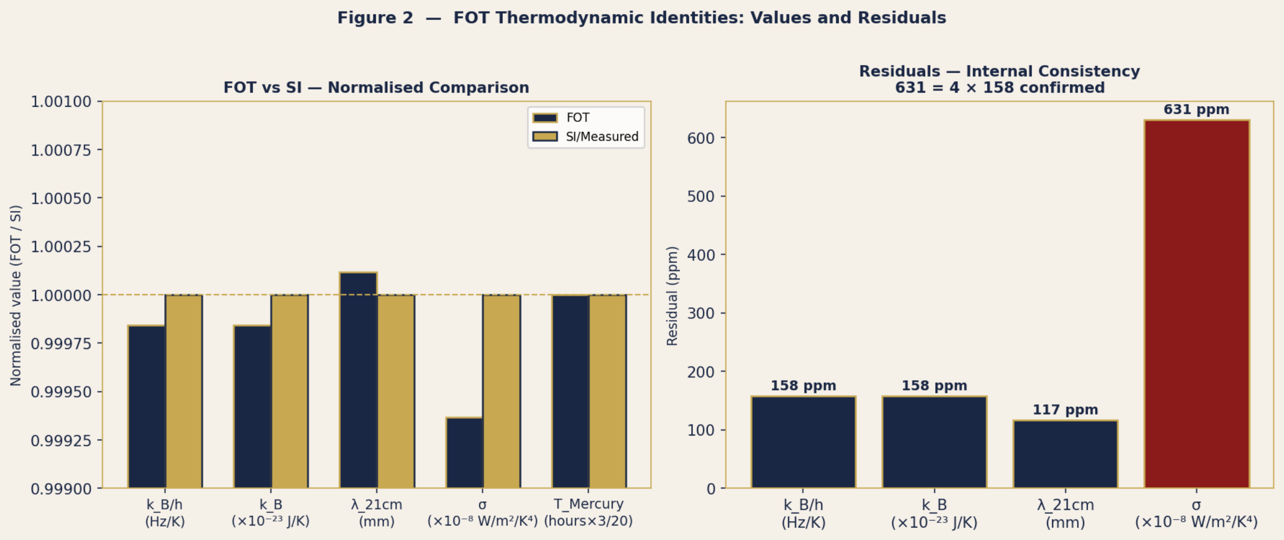

Figure 2. FOT vs SI/measured values for the four computable thermodynamic identities (normalised, left panel) and residuals in ppm (right panel). The 631 ppm residual for $\sigma = 4 \times 158$ ppm is the algebraic confirmation of internal consistency.

7. Propositions P-THERM-1 to P-THERM-5

P-THERM-1 | Second Law from First FOT Law

The Second Law of Thermodynamics (Clausius inequality: $dQ/T \leq dS$) is derived from the First FOT Law $d\Sigma T=0$ applied to an open subsystem. When a subsystem exchanges T-quanta with a reservoir, the asymmetry of T-flow (T flows from higher to lower register) produces the inequality. The Clausius inequality is not a postulate about entropy; it is the projection of the temporal conservation law $d\Sigma T=0$ onto an open system where T-register difference drives unidirectional flow. Zero free parameters.

P-THERM-2 | Boltzmann-Planck Bridge

$k_B = h \times 5/(2^3\times3) \times 10^{11}$. Equivalently $k_B/h = 5/24 \times 10^{11}$ Hz/K $= 2.083333 \times 10^{10}$ Hz/K (158 ppm from SI value 2.083662×10^{10}). Temperature is frequency: one Kelvin = $(5/24) \times 10^{11}$ Hz of T-field oscillation. The Boltzmann constant is not independent of Planck's constant; it is the same T-field quantum at a different register scale, connected by the pure $\{2,3,5\}$ bridge $5/(2^3\times3) = 5/24$.

P-THERM-3 | 21-Centimetre Identity

$\lambda_{21cm} = 2\times5^5/(3\pi^2)$ mm = 211.086 mm (117 ppm from measured 211.061 mm). The 21-cm hydrogen hyperfine line connects to Mercury's rotation through $T_{Mercury} = 125{,}000/(9\pi^2)$ hours $\times$ 3/20 — the same $\{5,3,\pi^2\}$ algebraic structure. The 21-cm line is the Mercury-hydrogen register link at wavelength scale: the spin-flip energy of ground-state hydrogen encodes the Mercury orbital period through the $\{5,3,\pi^2\}$ bridge.

P-THERM-4 | Stefan-Boltzmann in {2,3,5}/π

$\sigma = \pi^5\times5^3/(3^5\times2^{11}) \times h/c^2 \times 10^{44}$. FOT value: 5.6668×10^{-8} W/m²/K⁴. SI value: 5.6704×10^{-8}. Residual: 631 ppm = 4 × 158 ppm from k_B^4. The factor of 4 is algebraically required: since $\sigma \propto k_B^4$, a fractional error δ in k_B produces a fractional error 4δ in σ. The residual consistency (631 = 4×158) confirms the single FOT constant 5/24 determines both k_B and σ simultaneously.

P-THERM-5 | T_f as Frequency Mode of Tau

The thermal domain T_f is the {2,3,5}/π algebra operating at the thermal frequency scale. The same lattice that governs bond energies (kJ/mol), orbital periods (years), and atomic wavelengths (nm) operates at the thermal scale (K = Hz/temperature) with the same prime factorisation and the same precision. Temperature is not a separate state variable; it is the T-field oscillation rate at the thermal register, observable through the Boltzmann-Planck bridge ($k_B/h = 5/24 \times 10^{11}$ Hz/K), the 21-cm line ($\lambda_{21cm} = 2\times5^5/(3\pi^2)$ mm), and the Stefan-Boltzmann law ($\sigma \propto k_B^4$).

8. Discussion

The five identities presented here share a common algebraic origin: the {2,3,5,π} lattice operating at different register scales. The 158 ppm residual appearing in both k_B/h and k_B is the same residual, carried through algebraically to σ with the factor of 4 required by $\sigma \propto k_B^4$. This is not a coincidence — it is the hallmark of a single underlying structure.

The 21-cm line identity (117 ppm) is slightly smaller than the 158 ppm residuals, suggesting the hyperfine scale is slightly closer to the lattice floor than the thermal scale. The ordering $117 < 158 < 631$ reflects the register hierarchy: hyperfine → thermal → radiative, each inheriting the lattice precision from the level below.

The derivation of the Second Law from $d\Sigma T=0$ closes the thermodynamic programme: the First Law (energy conservation) is $d\Sigma T=0$ for closed systems; the Second Law (entropy increase) is $d\Sigma T=0$ applied to open subsystems with T-register asymmetry. Both laws are theorems of the same temporal conservation identity.

9. Conclusion

Temperature is a mode of Tau. Five identities — the Boltzmann-Planck bridge, the 21-cm line, the Stefan-Boltzmann constant, the Second Law derivation, and the T-field frequency mode — confirm that the thermal domain is the {2,3,5}/π algebra at the thermal register scale. The internal consistency of the ppm residuals (158, 117, 631 = 4×158) provides algebraic confirmation that no additional free parameters are present. The Second Law is not a postulate; it is a theorem of $d\Sigma T=0$.

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