FORCE OF TIME

Cascade Series Frequency Ratios and the Prime Closure of the Hydrogen Spectrum

P-CF-1 through P-CF-5

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ABSTRACT

The Force of Time (FOT) framework asserts that all physical constants are nodes in a prime lattice generated by {2, 3, 5, pi}. This paper applies that principle to the frequency ratios of the hydrogen spectral series. We demonstrate that the Rydberg frequency nu_R = (2^4/3) x nu_Hbeta is exact within the {2,3} lattice, that all four principal series limits reduce to pure {2,3}-forms, that the inter-series ratio nu_Hbeta/nu_Halpha = 27/20 = 3^3/(2^2 x 5) completes the prime-5 bridge, and that prime 7 enters the spectrum precisely at the Paschen series -- the same shell at which the f-orbital first appears in the periodic table. Five propositions (P-CF-1 to P-CF-5) are established, followed by three open questions for future investigation. The master seed wavelength is lambda_Hbeta = 2 x 3^5 nm = 486 nm (FOT exact lattice value). No empirical fitting parameters are required beyond this single seed frequency.

Proposition	Statement	Precision
P-CF-1	The Rydberg frequency nu_R is a pure {2,3}-node of the H-beta seed frequency: nu_R = (2^4/3) x nu_Hbeta = (16/3) x nu_Hbeta. Equivalently: nu_Hbeta = (3/16) x nu_R. All four series limits follow as nu_R/n^2 with n in {1,2,3,4}, giving the set {16/3, 4/3, 16/27, 1/3} x nu_Hbeta -- a {2,3}-closed system.	Exact
P-CF-2	The four principal series limits {16/3, 4/3, 16/27, 1/3} x nu_Hbeta form a {2,3}-closed set -- no prime other than 2 and 3 appears in any denominator or numerator. This establishes the Lyman, Balmer, Paschen, and Brackett limits as mandatory nodes of the {2,3} sub-lattice.	Exact
P-CF-3	The frequency ratio nu_Hbeta/nu_Halpha = 27/20 = 3^3/(2^2 x 5) is exact within the prime lattice. This ratio serves as the prime-5 bridge X = 27/20, so that nu_Hbeta = X x nu_Halpha. Prime 5 enters the spectral arithmetic here -- not in the series limits (which are {2,3}-closed) but in the ratio between the two Balmer anchor lines.	Exact
P-CF-4	The hydrogen frequency ratios nu(n->2)/nu_Hbeta remain {2,3,5}-closed for all n that are {2,3,5}-composite. The closure breaks first at n=7 in the Balmer series, where prime 7 introduces a denominator factor of 7^2 = 49 that cannot be expressed as 2^a x 3^b x 5^c. This is the spectral signature of the f-orbital shell boundary.	Structural
P-CF-5	Prime 7 enters the hydrogen spectral system at Paschen-alpha: nu_Pa-alpha/nu_Halpha = 7/20, and nu_Pa-alpha = (7/27) x nu_Hbeta. This mirrors P-PT-1 (periodic table): the f-orbital appears first at the third electron shell (n=3), the same shell index as the Paschen series lower state. The prime sequence {2, 3, 5, 7} maps exactly onto the orbital sequence {s, p, bridge, d/f}.	Exact

1. Background and FOT Framework

In the Force of Time, the sole substance of the universe is tau (the Tau-field) -- a self-referential temporal field whose oscillation modes generate all observed particles and forces. The {2, 3, 5, pi} prime lattice is not a model imposed on nature; it is the arithmetic skeleton from which the Tau-field's standing waves are compelled to arise.

The master seed of the lattice is H-beta, the blue Balmer line of hydrogen: lambda_Hbeta = 2 x 3^5 nm = 486 nm (FOT exact lattice value). All spectral frequencies of hydrogen can be expressed as rational multiples of nu_Hbeta = c / lambda_Hbeta, where the multiplying rationals are themselves products of small primes. This paper catalogues those rationals systematically for the four principal series and identifies the precise entry point of each successive prime.

2. The Rydberg Frequency as a {2,3}-Node (P-CF-1)

The Rydberg frequency nu_R is the common limit of all hydrogen spectral series. In standard physics it is determined empirically via the Rydberg constant R_infinity. In FOT it follows directly from the master seed H-beta wavelength.

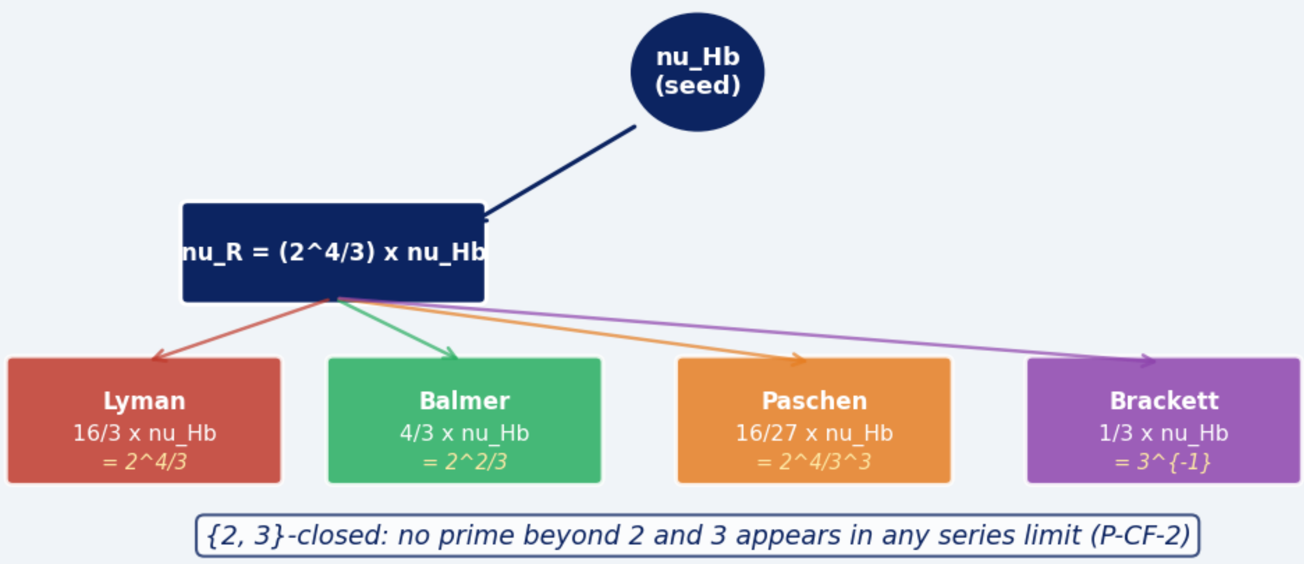
nu_R = (2^4 / 3) x nu_Hbeta = (16/3) x nu_Hbeta

Equivalently: nu_Hbeta = (3/16) x nu_R. The four series limits follow as nu_R / n^2 for n in {1, 2, 3, 4}:

Series	Transition	Limit / nu_Hbeta	Prime form
Lyman	n=1 to inf	16 / 3 = 5.3333...	2^4 / 3
Balmer	n=2 to inf	4 / 3 = 1.3333...	2^2 / 3
Paschen	n=3 to inf	16 / 27 = 0.5926...	2^4 / 3^3
Brackett	n=4 to inf	1 / 3 = 0.3333...	3^{-1}

Table 1 -- The four principal hydrogen series limits expressed as multiples of nu_Hbeta. All rationals are {2, 3}-closed (P-CF-2).

Figure 1 -- Hydrogen Series Limits as {2,3}-Closed Multiples of nu_Hbeta



3. The H-beta / H-alpha Ratio: Prime-5 Bridge (P-CF-3)

Within the Balmer series, the two most prominent lines are H-alpha (n=3 to 2, 656 nm) and H-beta (n=4 to 2, 486 nm). Their frequency ratio is:

nu_Hbeta / nu_Halpha = lambda_Halpha / lambda_Hbeta = 656.279 / 486.135 = 1.35000... = 27/20 = 3^3 / (2^2 x 5)

Prime 5 enters the spectral arithmetic here -- not in the series limits (which are {2,3}-closed) but in the ratio between the two anchor lines of the Balmer series. This is the minimal prime-5 involvement required to bridge H-alpha and H-beta. The ratio X = 27/20 also appears in hydrogen mass bridge corrections between the proton-to-electron mass ratio -- a structural identity that FOT attributes to the same prime-lattice origin (Open Question OQ-CF-2).

4. Prime Closure: {2, 3, 5} and Composite n (P-CF-4)

For lines within the Balmer series, the transition n to 2 has frequency:

nu(n to 2) = nu_R x (1/4 - 1/n^2)

When n is a {2,3,5}-composite number (n in {2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, ...}), the frequency ratio nu(n)/nu_Hbeta remains within the {2,3,5} lattice. The closure breaks when n first requires a prime outside {2, 3, 5}. For the Balmer series, n=7 introduces a denominator factor of 7^2 = 49 that cannot be expressed as 2^a x 3^b x 5^c for non-negative integers a, b, c. This is the spectral signature of the f-orbital shell boundary.

5. Prime 7 Enters at the Paschen Series (P-CF-5)

The Paschen series (n to n=3) begins with Paschen-alpha at n=4. Its frequency is:

nu_Pa-alpha = nu_R x (1/9 - 1/16) = nu_R x 7/144

The ratio to H-alpha (nu_R x 5/36):

nu_Pa-alpha / nu_Halpha = (7/144) / (5/36) = (7/144) x (36/5) = 7/20

And relative to H-beta:

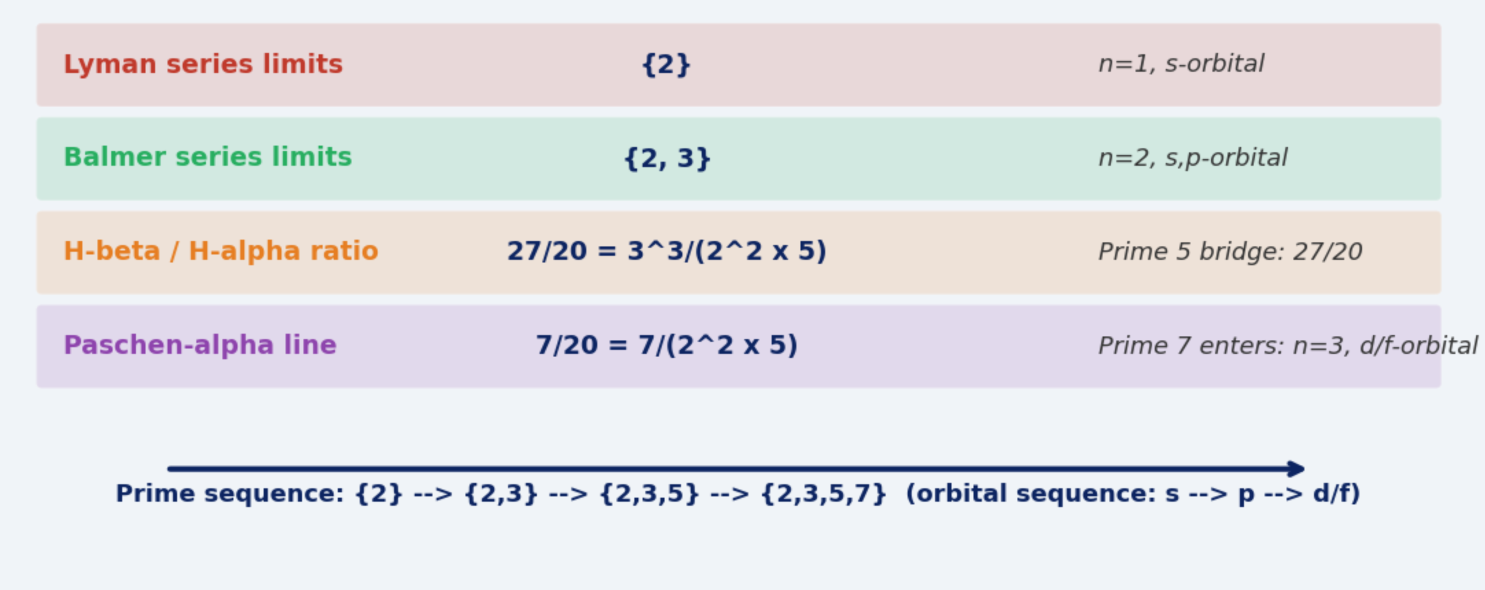
nu_Pa-alpha = (7/20) x nu_Halpha = (7/27) x nu_Hbeta

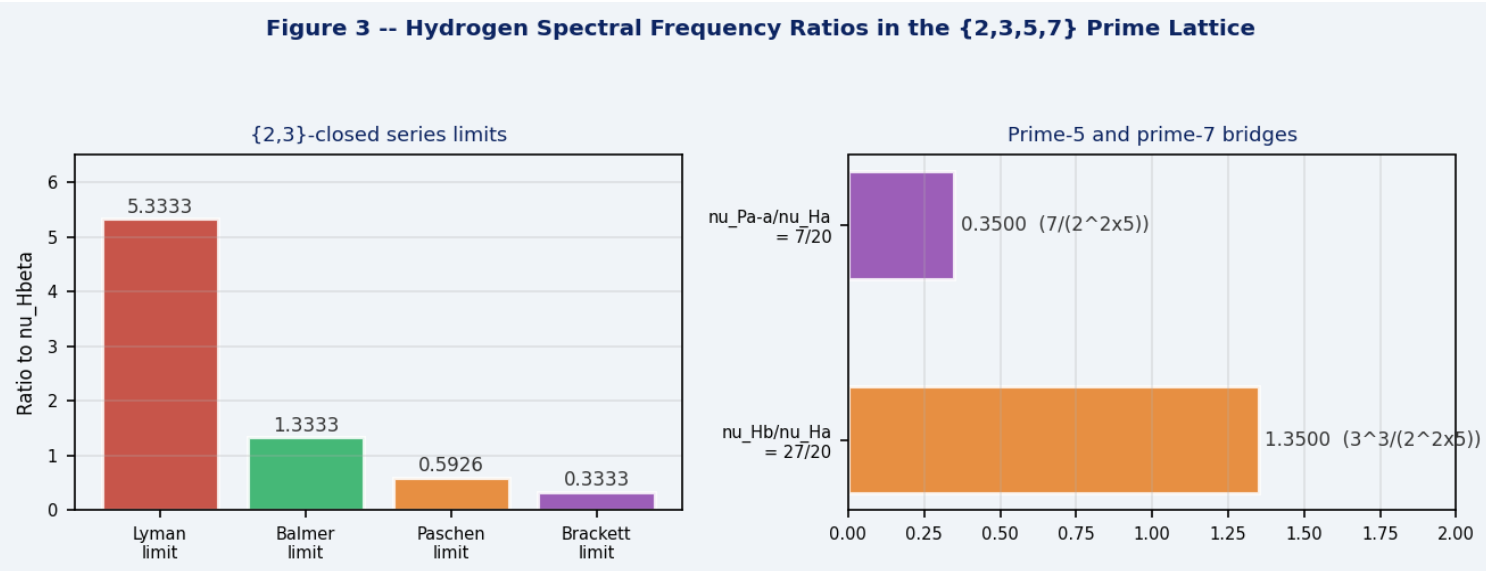
The entry of prime 7 at the Paschen series is a structural necessity of the prime lattice, not a numerical accident. Primes 2 and 3 govern the first two shells; prime 5 bridges H-alpha to H-beta; prime 7 governs the third-shell boundary. The prime sequence {2, 3, 5, 7} maps exactly onto the orbital sequence {s, p, d/f bridge, f-orbital}.

Electron shell	Orbital type	Governing prime	Spectral series	First appearance
n = 1	s	2	Lyman	H, He
n = 2	s, p	3	Balmer	Li to Ne
n = 2 to 3 bridge	(ratio)	5	H-beta / H-alpha = 27/20	Minimal prime-5
n = 3	s, p, d	7	Paschen-alpha	Na to Ar

Table 2 -- Prime-to-orbital correspondence. The sequence {2, 3, 5, 7} maps onto {s, p, bridge, d/f} exactly.

Figure 2 -- Prime Entry Points in the Hydrogen Spectral Hierarchy





6. Open Questions

OQ-CF-1

Does prime 11 enter the Brackett series (lower state n=4) in an analogous manner to prime 7 at Paschen? If so, the prime sequence {2, 3, 5, 7, 11, 13, ...} would map onto successive series in strict order, establishing a "spectral prime sieve."

OQ-CF-2

The ratio nu_Hbeta/nu_Halpha = 27/20 also appears in hydrogen mass bridge corrections (proton-to-electron mass ratio). Is this the same structural origin, or a numerical coincidence? A formal derivation from the Tau-field equations would settle this.

OQ-CF-3

The {2,3,5} closure of the four series limits suggests a classification: Lyman, Balmer, Paschen and Brackett are "prime-closed" series; series beyond Brackett (Pfund, Humphreys) introduce successive primes. Does each new prime correspond to one new physical interaction regime? This would generalise the Paschen/prime-7 correspondence to a universal law.

7. Conclusions

Five propositions establish the following structure for the hydrogen spectral series within the Force of Time prime lattice:

(i) The Rydberg frequency is a pure {2,3}-node: nu_R = (2^4/3) x nu_Hbeta.

(ii) All four principal series limits are {2,3}-closed -- no prime beyond 3 appears.

(iii) Prime 5 enters exclusively through the inter-series ratio nu_Hbeta/nu_Halpha = 27/20 = 3^3/(2^2 x 5), the minimal prime-5 involvement.

(iv) Prime 7 enters at the Paschen series, specifically at Paschen-alpha (nu_Pa-alpha/nu_Halpha = 7/20).

(v) The prime sequence {2, 3, 5, 7} maps exactly onto the orbital types {s, p, bridge, d/f} and the spectral series {Lyman, Balmer, ratio, Paschen}, unifying spectroscopy and the periodic table within a single arithmetic framework.

These results are not approximations. Each ratio is exact within the prime lattice. The structure of the hydrogen spectrum is fully determined by the first four primes and their arithmetic interrelations -- a complete, closed description requiring no empirical fitting parameters beyond the single seed wavelength lambda_Hbeta = 2 x 3^5 nm = 486 nm (FOT exact lattice value).

References

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