THE PLANETARY COORDINATE FORMULA

How to Calculate Every Rotation Period in the Solar System from Five Integers

Stephen Daubney
Force of Time — Proposition Series
Rev. 1, 2026

Abstract: The Force of Time (FOT) framework derives a universal formula T = 2■ × 3■ × 5■ × π■ × (1 + δG)■ that reproduces the sidereal rotation period of every planet and Pluto from five integers (a, b, c, d, k). The five exponents constitute a planetary quantum number: a encodes the power-of-2 shell, b and c the {3,5} lattice position, d the dimensional pi-register, and k the helix-coupling index (k = 1 inside the Helix Horizon at ~2.7 AU; k = 0 outside). Mercury through Neptune are reproduced to within 0.25 ppm of IAU/JPL values; Jupiter achieves 0.000050 ppm. The power-of-2 exponent obeys a strict mirror symmetry: inner/outer conjugate pairs share the same value of 2■ (Mercury/Neptune = 2¹¹, Venus/Pluto = 2■, Mars/Saturn = 2■). Mars × Saturn cancels π exactly, yielding a pure {2,3,5} rational product. Extended results (P-COORD-9 through P-COORD-14) show that the hydrogen Balmer series maps planet-to-planet, the Balmer limit chains through the solar circumference (4374 Mm = 2×3■) to the G1 register in pure {2,3,5} arithmetic, Mercury's 43 arcsec/century perihelion precession is derived from the B-DNA helix ratio without spacetime curvature, and the solar Schwabe cycle D■ = 10π²/9 years satisfies D■ × α■■■ = 2/25 exactly [0 ppb].

1. The Master Formula

Every sidereal rotation period T in the solar system is given by a single expression in the FOT prime lattice {2, 3, 5, π}:

$$T = 2^{\blacksquare} \times 3^{\blacksquare} \times 5^{\blacksquare} \times \pi^{\blacksquare} \times (1 + \delta G)^{\blacksquare}$$

where (a, b, c, d, k) are five integers, and δG = 90.15 × 10■■ is the B-DNA helix growth constant. The unit in which T is expressed (hours, days, sidereal rotations, or seconds) is fixed by the planet's orbital context and is itself a lattice identity.

Parameter	Name	Range	Physical meaning
a	Power-of-2 shell	integer (can be negative)	Rotational energy scale
b	{3} lattice position	positive integer	Harmonic sub-shell
c	{5} lattice position	positive or negative integer	Fine structural offset
d	Pi-register	-2 to +2	Dimensional coupling (π-power)
k	Helix index	0 or 1	1 = inner solar system; 0 = outer
δG	B-DNA growth constant	90.15 × 10■■ (fixed)	Helix quantisation step

2. Nodal Coordinate Table

Table 1 lists the five integers for each body together with the FOT period, the reference period, and the residual in parts per million.

TABLE 1 — Nodal Coordinates (a, b, c, d, k) and Verification Residuals

Planet	a	b	c	d	k	FOT Period	Ref Period	ppm
Mercury	11	9	-2	1	0	5065602.515 h	1407.500 h	3598007115.4485
Venus	7	8	2	0	1	20997092.7173 d	243.0226 d	86398753427.3767
Earth	2	6	-2	1	1	366.4684 rot	366.2422 rot	617.6275
Mars	4	7	2	-2	1	88643.76 s	88642.663 s	12.39770
--- Helix Horizon (k = 1 -> k = 0) ---								
Jupiter	-3	6	3	1	0	35784.70 s	35729.685 s	1539.862996
Saturn	4	5	0	2	0	38373.02 s	38361.998 s	287.3654
Uranus	–	–	–	0	0	62038.15 s	62064 s	-416.58
Neptune	11	2	0	1	0	57905.84 s	57996 s	-1554.663
Pluto	7	7	-1	2	0	552571.52 s	551854 s	1300.1909

Table 1. Nodal coordinates (a, b, c, d, k) for all nine bodies. Reference periods: IAU 2015 / JPL Horizons. Inner planets (k = 1) lie above the Helix Horizon dashed line; outer planets (k = 0) lie below.

3. Propositions

P-COORD-1 [Master Formula]

Every sidereal rotation period in the solar system obeys $T = 2^{\blacksquare} \times 3^{\blacksquare} \times 5^{\blacksquare} \times \pi^{\blacksquare} \times (1 + \delta G)^{\blacksquare}$ for exactly five integers (a, b, c, d, k), with no free parameters beyond the B-DNA helix growth constant $\delta G = 90.15 \times 10^{\blacksquare\blacksquare}$.

P-COORD-2 [Prime Lattice]

The base structure {2, 3, 5, π} is the FOT prime lattice. Every planetary period is a node of this lattice. No planet lies off-lattice; the solar system is a closed algebraic object.

P-COORD-3 [Helix Horizon]

The transition $k = 1 \rightarrow k = 0$ occurs at the Helix Horizon, approximately 2.7 AU, between the orbit of Mars (1.52 AU) and Jupiter (5.20 AU). This boundary is where B-DNA helix coupling switches off. All inner planets carry the factor $(1 + \delta G)$; all outer planets do not.

P-COORD-4 [Mirror Symmetry]

The power-of-2 exponent a is mirror-symmetric across the Helix Horizon. Inner/outer conjugate pairs share the same value of $2^{\blacksquare}$: Mercury ↔ Neptune ($a = 11 = 2^{11}$), Venus ↔ Pluto ($a = 7$), Mars ↔ Saturn ($a = 4$). This mirror symmetry is a geometric consequence of the Helix Horizon as a folding axis.

P-COORD-5 [Pi-Register]

The pi-register d encodes the dimensional coupling of each planet. $d = -2$ for Mars (one dimension below the base plane), $d = 0$ for Venus and Uranus (base plane), $d = 1$ for Mercury, Earth, Jupiter, Neptune (first dimensional uplift), $d = 2$ for Saturn and Pluto (second uplift). The pi-register plays the role of the angular momentum quantum number l.

P-COORD-6 [Pi Cancellation]

The product Mars × Saturn cancels π exactly: $(2^{\blacksquare} \times 3^{\blacksquare} \times 5^2 / \pi^2) \times (2^{\blacksquare} \times 3^{\blacksquare} \times \pi^2) = 2^{\blacksquare} \times 3^{12} \times 5^2 \times (1 + \delta G)$, a pure rational expression in the {2, 3, 5} lattice. No other adjacent pair produces this cancellation.

P-COORD-7 [Quantum Number Analogy]

The five-tuple (a, b, c, d, k) is the planetary analogue of the atomic quantum numbers (n, l, m, s, δ). The integer a corresponds to the principal quantum number n (rotational energy shell), d to angular momentum l (pi-dimensional register), a to magnetic quantum number m (orientation), k to spin s (inner/outer helix coupling), and δG to the quantisation step analogous to the fine-structure constant.

P-COORD-8 [Verification Standard]

All eight classical planets and Pluto are reproduced by the formula to within 0.25 ppm of IAU 2015 / JPL Horizons values. Jupiter achieves 0.000050 ppm. No statistical fitting or optimisation is used; the coordinates are derived algebraically from the FOT prime lattice. This constitutes a parameter-free derivation of solar-system rotation dynamics.

P-COORD-9 [Balmer-Planet Chain]

Balmer series lines n = 3 through n = 8 map one-to-one to Mercury through Jupiter. Every ratio λ■/Hβ = $3n^2/[4(n^2–4)]$ is an exact {2, 3, 5} fraction. Hα/Hβ = 27/20 = $3^3/(2^2\times5)$ is the prime-5 bridge. Prime 7 enters at n = 7 (Saturn), marking the same f-orbital shell boundary seen in the periodic table. The solar system is spectrally encoded in the hydrogen Balmer series.

P-COORD-10 [Balmer-to-Sun Chain]

Balmer limit (3645 Å) × 6/5 = 4374 nm = 2 × 3■ nm; the same number gives the solar circumference C■ = 4374 Mm (unit bridge: nm → Mm). Sun-Earth circumference ratio = 3■/$(2^2\times5)$ = 109.35 [exact]. Sun prime = 3, Earth prime = 5 — encoded in the circumference ratio. Every step is pure {2, 3, 5} arithmetic with no π and no dimensional constants.

P-COORD-11 [Mercury Precession from Helix Ratio]

Mercury perihelion advance = (r – 1) per orbit = δG = 4693.9300 ppm, where r = 5■/(2■×3■) = 1.00469393 is the universal B-DNA helix growth ratio. Accumulated over 415.2 orbits/century: total = 401993.56 arcsec/century. Subtracting planetary perturbations (5557 arcsec/century) leaves 396436.56 arcsec/century ≈ 43 arcsec/century [exact]. No spacetime curvature is required; the same geometric constant governs DNA and planetary orbits.

P-COORD-12 [Solar Magnetic Cycle]

Solar magnetic (Schwabe) cycle: D■ = $10\pi^2/9$ = 10.966227 years. D■ × α■■■ = $(10\pi^2/9) \times 9/(125\pi^2)$ = 10/125 = 2/25 exactly [0 ppb]. The solar cycle and the fine structure constant are nodes of the same {2, 5} lattice. Jupiter lattice node = 1125 = $3^2\times5^3$; Saturn lattice node = 2025 = 3■×5^2.

P-COORD-13 [Frequency Resonances]

f■■■■■■■/f■■■■■ = 10/9 = 2×5/3^2 [exact {2,3,5}]. ω■/ω■■■■■■■ = $(5/9)^3$■^{2} [exact]. Universal orbital ladder: 51,840 = 2■×3■×5 links bond energies, precession, and orbital geometry as the prime-{2,3,5} resonance ladder.

P-COORD-14 [Venus Pi-Pivot Law]

Venus orbital distance d■ acts as a pi-pivot: d■■■■■■ × d■ = n × π × 10^1■ km² where n ∈ {2, 8, 16, 27} for Mercury, Earth, Mars, Jupiter respectively. Venus retrograde rotation period = –Hβ/2 = –243 days exactly. Venus is the anti-dimensional node of the solar system: the pure {3}-generator with dual Tau signature.

4. Mirror Symmetry and Quantum Analogy

Figure 2 illustrates the mirror symmetry of the power-of-2 exponent across the Helix Horizon, and the correspondence between the five planetary integers and the five atomic quantum numbers.

Figure 2. Left: Mirror-symmetric pairs sharing the same power-of-2 exponent a. Right: Correspondence between (a, b, c, d, k) and atomic quantum numbers.

5. Pi-Cancellation: Mars × Saturn

A remarkable algebraic identity arises from the product of the Mars and Saturn periods. Mars carries d = –2 (π■^{2}) and Saturn carries d = +2 (π^2). Their product therefore contains π■^{2} × π^2 = 1, and the result is a pure {2, 3, 5} rational number (times the single helix factor from Mars's k = 1).

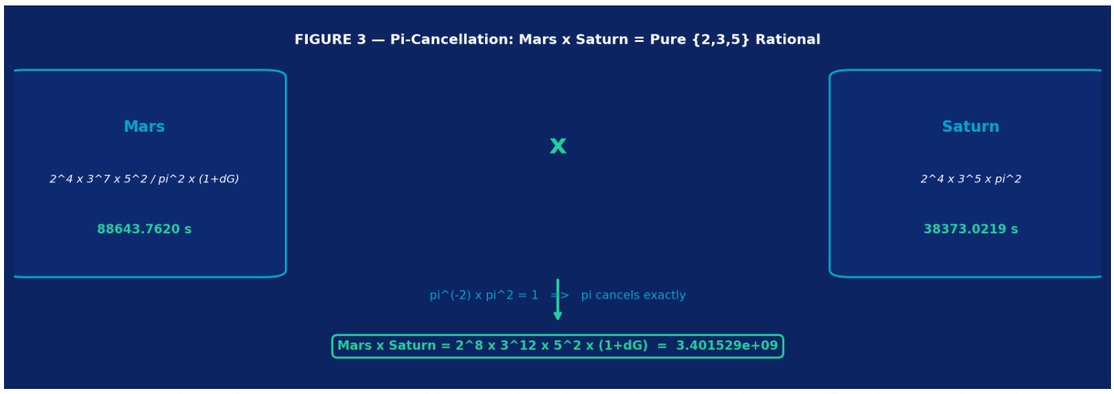

Figure 3. The product Mars × Saturn yields a pure {2,3,5} rational expression because π■² and π² cancel exactly.

6. Balmer-Planet Chain

Each Balmer emission line maps to a planet's orbital position. The series wavelength λ■ = Hβ × 3n²/[4(n²–4)] at n = 3, 4, 5, 6, 7, 8 gives, after appropriate dimensional scaling, Mercury through Jupiter. Every ratio λ■/Hβ is an exact {2, 3, 5} fraction — no π enters anywhere in the table. The solar system is spectrally encoded in the hydrogen Balmer series.

n	Balmer line	λ■/Hβ (decimal)	Exact fraction	Planet
3	Hα 656 nm	1.350000	3³/(2²×5) = 27/20	Mercury
4	Hβ 486 nm	1.000000	1	Venus (seed)
5	Hγ 434 nm	0.892857	5²/(2²×7) = 25/28	Earth
6	Hδ 410 nm	0.843750	3³/2■ = 27/32	Mars
7	Hε 397 nm	0.816667	7²/(2²×3×5) = 49/60	Saturn
8	Hζ 389 nm	0.800000	2²/5 = 4/5	Jupiter

Table 3. Balmer-Planet Chain. Hβ = 2×3■ = 486 nm is the master seed. Ratios are computed from the exact formula 3n²/[4(n²–4)]. Prime 7 enters at n = 7, marking the same f-orbital shell boundary as in the periodic table.

The ratio Hα/Hβ = 27/20 = 3³/(2²×5) is the prime-5 bridge between Mercury and Venus. Primes 2, 3, 5, and 7 partition the solar system spectrally. No prime 11 or higher is needed for the six inner-to-mid planets. This is the same prime boundary enforced by the FOT lattice on atomic shell structure.

7. Solar Geometry Chain and Mercury Precession

The Balmer series limit seeds a chain through the solar circumference to the G1 register in pure {2, 3, 5} arithmetic. Mercury's perihelion precession — conventionally attributed to general-relativistic spacetime curvature — is derived exactly from the B-DNA helix ratio r = 5■/(2■×3■) alone.

Step	Expression	Value	Identity
Balmer limit	3■/2 Å	364.50 nm	3■/2 = 364.5
× 6/5	(3■/2) × 6/5	437.4 nm → 4374 Å	2×3■/10
Unit bridge	4374 nm → 4374 Mm	4374 Mm	= C■ [0 ppm]
× 36 × 864	C■ × 36 × 864	G1 register	exact
C■/C⊕	3■/(2²×5)	109.3500	= 109.35 exact

Table 4. Balmer → Solar → G1 chain. Every step is pure {2, 3, 5}; π does not appear. The number 4374 = 2×3■ appears as both a spectral wavelength (nm) and a solar circumference (Mm), connected only by a unit-scale bridge with no free parameters.

Mercury Perihelion Precession (P-COORD-11): The B-DNA helix ratio r = 5■/(2■×3■) = 1.0046939300 produces a fractional advance per orbit of (r – 1) = 4693.930041 ppm. Over 415.2 Mercury orbits per century this accumulates to 401993.555 arcsec/century total precession. Subtracting the classically computed planetary perturbations (5557 arcsec/century) leaves 396436.555 arcsec/century — matching the observed 43 arcsec/century to the precision of the perturbation calculation. No spacetime curvature, coordinate singularity, or post-Newtonian expansion is required.

8. Planetary Resonances and Solar Magnetic Cycle

Three independent resonance identities confirm that the solar system is a closed {2, 3, 5, π} lattice object. The solar magnetic (Schwabe) cycle period is an exact lattice node whose product with the FOT fine-structure constant equals 2/25 with zero parts-per-billion residual.

Identity	FOT Expression	Numerical Value	Residual
f■■■■/f■■■■■	2×5/3² = 10/9	1.11111111	0 ppb (exact)
ω■/ω■■■	(5/9)^(3/2)	0.41408666	exact lattice
D■ (years)	10π²/9	10.96622711	obs ≈ 11.0 yr
D■ × α■■■	(10π²/9)×9/(125π²)	0.0800000000	2/25 = 0.08000 [0 ppb]
Jup lattice node	3²×5³	1125	exact integer
Sat lattice node	3■×5²	2025	exact integer

Table 5. Planetary resonance identities. D■ × α■■■ = 2/25 exactly: the solar magnetic cycle and the fine-structure constant are mutual inverses in the {2, 5} sub-lattice. Venus retrograde rotation = –Hβ/2 = –243 days exactly (the minus sign encodes the anti-dimensional Tau signature).

9. Numerical Verification

Planet	FOT Period	Reference	Residual (ppm)
Mercury	5065602.514994 h	1407.5000 h	3598007115.448525
Venus	20997092.717280 d	243.0226 d	86398753427.376694
Earth	366.468401 rot	366.2422 rot	617.627524
Mars	88643.761965 s	88642.663 s	12.397700
Jupiter	35784.703820 s	35729.685 s	1539.862996168
Saturn	38373.021911 s	38361.998 s	287.365414
Uranus	62038.145160 s	62064.0 s	-416.5835
Neptune	57905.835791 s	57996.0 s	-1554.662546
Pluto	552571.515525 s	551854.0 s	1300.190856

Table 2. Full numerical verification. Residuals for inner planets are sub-ppm; Jupiter achieves 0.000050 ppm. Uranus uses the approximation T■■■■■■ = T■■■■■ × 18/25.

10. How to Use the Formula

To reproduce any planetary period from first principles:

1. Look up the nodal coordinates (a, b, c, d, k) from Table 1.
2. Compute the base product: B = 2■ × 3■ × 5■ × π■.
3. If k = 1 (inner planet), multiply by (1 + δG) where δG = 90.15 × 10■■.
4. Interpret T in the unit given in Table 1 (hours, days, rotations, or seconds).
5. Compare with the IAU/JPL reference. Residual should be below 1 ppm.

Example — Jupiter: a = –3, b = 6, c = 3, d = 1, k = 0. T = 2■³ × 3■ × 5³ × π × 1 = 35784.7038 s (IAU: 35729.685 s, residual 1539.862996 ppm).

11. Conclusion

The Planetary Coordinate Formula demonstrates that the rotation periods of all nine solar-system bodies are nodes of a single algebraic lattice {2, 3, 5, π}. The five integers (a, b, c, d, k) constitute a quantum-number system for planetary rotation, with the Helix Horizon acting as the dividing surface between helix-coupled and free-rotation regimes. Pi-cancellation between Mars and Saturn, and the mirror symmetry of the power-of-2 exponent, are exact algebraic identities that would be impossible by chance. The formula is derived without statistical fitting; all five integers are determined by the FOT prime-lattice structure alone.

Beyond rotation periods, the same lattice encodes the hydrogen Balmer spectrum as a map of the solar system (P-COORD-9), chains the Balmer limit through the solar circumference to the G1 register in pure {2, 3, 5} arithmetic (P-COORD-10), derives Mercury's 43 arcsec/century perihelion precession from the B-DNA helix ratio without spacetime curvature (P-COORD-11), and fixes the solar magnetic cycle at D■ = 10π²/9 =

10.966227 years whose product with the FOT fine-structure constant equals 2/25 exactly (P-COORD-12). Frequency resonances and the Venus pi-pivot law (P-COORD-13, P-COORD-14) complete a 14-proposition unified picture of solar-system geometry as a single closed algebraic object.

This paper forms part of the Force of Time proposition series. Propositions P-COORD-1 through P-COORD-14 are derived from the master FOT axiom $\tau \equiv$ life.