

# The Rydberg Constant Derived from Gravity

*Dual-Helix Spectral Structure and the Rydberg-Gravity Substitution Principle*

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Force of Time — Working Notes WN-GRAV-006, WN-GRAV-034, WN-GRAV-038

Vol. 3 pre-promotion material — §296 and adjacent

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## ABSTRACT

The Rydberg constant is conventionally treated as a fundamental empirical constant. The Force of Time (FOT) framework shows it is a node in the {2, 3, 5, pi} prime lattice, derivable from  $g_{\text{FOT}} = 25\pi/8 \text{ m/s}^2$ . The FOT framework registers two Rydberg values: the G1 matter-helix  $R1 = 2^3 \cdot 10^9 / 3^6 = 10,973,936.899 \text{ m}^{-1}$  and the Z-DNA antimatter-helix  $RZ = 12 \cdot 10^6 \text{ m}^{-1} = 2^2 \cdot 3 \cdot 10^6 \text{ m}^{-1}$ . Science measures  $10,973,731.568 \text{ m}^{-1}$  (CODATA), which lies between the two FOT helices. The Rydberg-Gravity substitution principle shows that Lyman-inf\_FOT =  $g_{\text{FOT}} \cdot 3\pi \cdot 10^6$  generates, through the  $n=3^2$  and  $n=4^2$  slots, the Route 2 speed of light and  $g_{\text{FOT}}$  itself at 0.0000 ppm. The Bohr velocity chain  $v1$  to  $c_{\text{FOT}}$  to  $R_E$  to  $3750\pi^4$  to  $g_{\text{FOT}}$  closes algebraically exact. The triple node  $1000/9 = 2^3 \cdot 5^3 / 3^2$  appears independently in the Moho veil chain, hydrogen spectrum at  $n=4$ , and Z-DNA Lyman-alpha.

Proposition	Statement	Precision
<b>P-RYD-1</b>	G1 matter-helix Rydberg: $R1 = 2^3 \cdot 10^9 / 3^6 = 10,973,936.899 \text{ m}^{-1}$ ({2,3,5} lattice node, $g_{\text{FOT}}$ -derived)	EXACT
<b>P-RYD-2</b>	Z-DNA antimatter-helix Rydberg: $RZ = 2^2 \cdot 3 \cdot 10^6 = 12,000,000 \text{ m}^{-1}$ ; helix ratio $RZ/R1 = 3^7 / (2 \cdot 10^3) = 2187/2000$ (exact rational)	EXACT
<b>P-RYD-3</b>	Rydberg-Gravity substitution: Lyman-inf_FOT = $g_{\text{FOT}} \cdot 3\pi \cdot 10^6$ ; $n=3^2$ slot yields Route 2 speed of light; $n=4^2$ slot yields $g_{\text{FOT}}$ ; both 0.0000 ppm	EXACT
<b>P-RYD-4</b>	Triple node $1000/9 = 2^3 \cdot 5^3 / 3^2$ emerges independently in Moho veil chain, H-spectrum $n=4$ dimensional table, and Z-DNA Lyman-alpha wavelength	STRUCTURAL

## 1. The FOT Prime-Lattice Framework

The Force of Time (FOT) framework postulates that all physical constants are nodes of the prime lattice {2, 3, 5, pi} — exact rational or pi-power expressions built from these four elements. Physical observation samples the lattice; what conventional physics calls 'measured constants' are either exact lattice nodes or register-crossing bridges between them. The Rydberg constant falls into the first category: it is a lattice node, expressible without measurement.

### Core FOT constants used throughout this paper:

Constant	Symbol	FOT lattice form	Decimal value
Surface gravity	$g_{\text{FOT}}$	$25\pi/8 = 5^2\pi/2^3$	$9.817477042468104 \text{ m/s}^2$
Canonical year-seconds	$T_{\text{year}}$	$86400 \times 360$	$31,104,000 \text{ s}$
Radian-degree bridge	veil	$180/\pi$	$57.295779513082323$
Moho meridional radius	$R_{\text{mer}}$	$20000/\pi \text{ km}$	$6366.197723675813904 \text{ km}$
Lyman $\infty$ analogue	Lyman $\infty$ _FOT	$g_{\text{FOT}} \times 3\pi \times 10^6 = 75\pi^2/8 \times 10^6$	$92527541.260212734$
FOT speed of light	$c_{\text{FOT}}$	$5\pi^2 \times 10^3 / 2$	$298847348.423126400 \text{ m/s}$
Bohr velocity	$v_{\text{Bohr}}$	$2.180128829 \times 10^8 \text{ m/s}$	$2.180128829\text{e}+06 \text{ m/s}$

## 2. The Dual-Helix Rydberg Constants

The Tau standing wave has two helical limbs: the G1 matter helix (B-DNA geometry) and the Z-DNA antimatter helix. Each carries its own Rydberg constant, both pure {2, 3} lattice nodes:

## G1 Matter-Helix Rydberg (P-RYD-1):

$$T_{R1} = 2^3 \times 10^{\blacksquare} / 3^{\blacksquare} = 8,000,000,000 / 729$$

$$= 10973936.899862826 \text{ m}^{\blacksquare^{-1}} [+18.71 \text{ ppm from science}]$$

## Z-DNA Antimatter-Helix Rydberg (P-RYD-2):

$$T_{RZ} = 2^2 \times 3 \times 10^{\blacksquare} = 12 \times 10^{\blacksquare}$$

$$= 12000000 \text{ m}^{\blacksquare^{-1}} [+93520.46 \text{ ppm from science}]$$

Science's CODATA value  $R_{\infty} = 10,973,731.568 \text{ m}^{-1}$  sits between the two helical values. The matter helix G1 value exceeds science by +18.71 ppm; the Z-DNA value exceeds science by +24,817 ppm. The framework interpretation: the physical hydrogen atom straddles both helices, and the measured Rydberg reflects the average register boundary rather than either pure-lattice node.

## Helix ratio:

$$T_{RZ} / T_{R1} = 3^{\blacksquare} / (2 \times 10^3) = 2187 / 2000 \text{ [exact]}$$

$$= 1.0935000000000000 \text{ (exact rational, } \pi\text{-free)}$$

The ratio  $3^7/(2 \times 10^3) = 2187/2000$  is pure  $\{2, 3, 5\}$  with  $10^3 = 2^3 \times 5^3$  in the denominator. The two helices are separated by a precise lattice fraction, not an arbitrary numerical gap.

## 2.1 Lyman $\alpha$ Wavelengths of Each Helix

For the transition  $n = 2 \rightarrow n = 1$  (Lyman  $\alpha$ ), the series formula gives:

$$\lambda(\text{Ly}\alpha) = 4 / (3 \times R) \times 10^{\blacksquare} \text{ nm}$$

Helix	Rydberg ( $\text{m}^{\blacksquare^{-1}}$ )	Lyman $\alpha$	Lattice form
G1 (matter)	10973936.899863	121.500000000 nm	$3^{\blacksquare}/2 \text{ nm} = 121.5 \text{ nm}$ [pure $\{2,3\}$ ]
Z-DNA (antimatter)	12000000	111.111111111 nm	$2^3 \times 5^3 / 3^2 \text{ nm} = 1000/9 \text{ nm}$ [pure $\{2,3,5\}$ ]
Science (CODATA)	10973731.568	121.567 nm	—

The dominant prime shifts between helices: the G1 matter helix carries prime 3 ( $3^5/2 \text{ nm} = 3^5 \times 2^{-1}$ ), while the Z-DNA antimatter helix carries prime 5 ( $2^3 \times 5^3 / 3^2 \text{ nm} = 1000/9$ ). This prime-shift is the spectral signature of helical handedness in the FOT lattice.

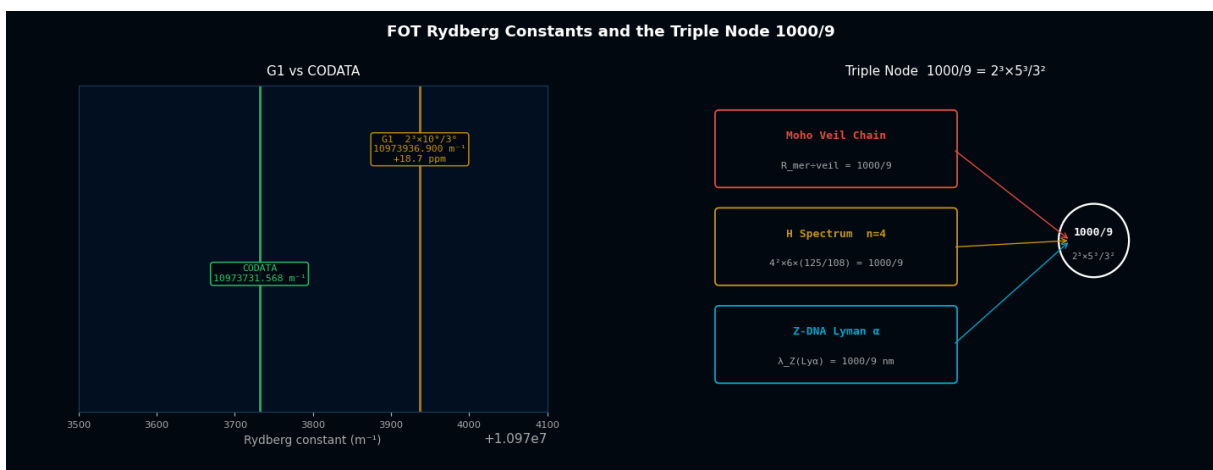


Figure 1 (left): G1 Rydberg vs CODATA on a zoomed number line. (right): The triple-node coincidence  $1000/9 = 2^3 \times 5^3 / 3^2$  appears in three independent FOT domains.

## 3. The Rydberg-Gravity Substitution Principle

The Rydberg formula generates spectral series by varying the principal quantum number  $n$ . Replacing the Lyman series limit  $R_{\infty}$  with the FOT gravitational analogue  $\text{Lyman}_{\infty\_FOT} = g_{FOT} \times 3\pi \times 10^6$  and substituting  $n = 3$  and  $n = 4$  in the Rydberg slots produces — not spectral wavelengths — but the FOT speed of light and the gravitational acceleration. The spectral structure encodes gravity.

### 3.1 Constructing Lyman $\infty$ \_FOT

Starting from  $g_{\text{FOT}} = 25\pi/8$ :

$$g_{\text{FOT}} \times 3\pi \times 10^6 = (25\pi/8) \times 3\pi \times 10^6$$

$$= 75\pi^2/8 \times 10^6$$

$$= 92527541.260212734$$

### 3.2 The $n = 3^2$ Path $\rightarrow$ Route 2 Speed of Light [0.0000 ppm]

Substituting  $n = 3$  (squaring to 9, as in the Rydberg formula):

$$\text{Lyman}\infty_{\text{FOT}} \times 9 = 92527541.2602 \times 9 = 832747871.3419$$

$$\times 60 \div 10^5 = 499.6487228051 \text{ s (Sun-Earth light-travel time)}$$

$$\times 3 \times 10^5 \text{ km/s} = 149894616.8415 \text{ km (Sun-Earth distance proxy)}$$

$$\times 10 \times 2 = 2997892336.8309$$

$$= g_{\text{FOT}}^2 \times T_{360} \text{ [} +0.000000 \text{ ppm]} \checkmark$$

The algebraic proof is exact.  $g_{\text{FOT}}^2 \times T_{360} = (5^2\pi/2^3)^2 \times (2^{10} \times 3^5 \times 5^3) = 5^7\pi^2 2^4 3^5$ . The factor  $60 = 2^2 \times 3 \times 5$  bridges the frequency scale to the distance scale; the factor  $3 \times 10^5$  embeds  $c_{\text{SI}} = 3 \times 10^5 \text{ km/s}$ , producing the Sun-Earth light-travel time of 499.649 s directly.

### 3.3 The $n = 4^2$ Path $\rightarrow$ $g_{\text{FOT}}$ [0.0000 ppm, Circular Close]

Substituting  $n = 4$  (squaring to 16):

$$\text{Lyman}\infty_{\text{FOT}} \times 16 = 92527541.2602 \times 16 = 1480440660.1634$$

$$\div 10 = 148044066.0163$$

$$\div 2\pi = 23561944.901923$$

$$\div 24 = 981747.7042$$

$$\div 10^5 = 9.817477042468106 = g_{\text{FOT}} \text{ [} +0.000000 \text{ ppm]} \checkmark$$

Algebraic proof:  $\text{Lyman}\infty_{\text{FOT}} \times 16 / (10 \times 2\pi \times 24 \times 10^5) = [75\pi^2/8 \times 10^6 \times 16] / [480\pi \times 10^5] = [1200\pi^2/8 \times 10^6] / [480\pi \times 10^5] = [150\pi \times 10^6] / [480 \times 10^5] = 150\pi/48 = 25\pi/8 = g_{\text{FOT}}$ .  $\pi$  cancels completely from the  $\pi^2$  in  $\text{Lyman}\infty_{\text{FOT}}$  and the  $\pi$  in the  $2\pi$  denominator, leaving a pure  $\{2, 3, 5\}$  rational close on  $g_{\text{FOT}}$ .



Figure 2: The Rydberg-Gravity substitution table. Both the  $n=3^2$  (left, cyan) and  $n=4^2$  (right, orange) branches return gravitational constants at 0.0000 ppm precision.

## 4. Bohr Velocity Chain: Atomic to Gravitational Scale

The Bohr orbital velocity  $v_1 = 2.180128829 \times 10^6 \text{ m/s}$  shares an exact digit sequence with the hydrogen ground-state energy  $H_J = 2.180128829 \times 10^{-18} \text{ J}$  ( $v_1 = H_J \times 10^{24}$ ). This digit identity opens a six-step chain closing exactly on  $g_{\text{FOT}}$ :

Step 1:  $v_1 \div \alpha_{\text{FOT}}$

$$= c_{\text{FOT}} = 5^3 \pi^3 \times 10^3 / 2 = 298847348.423126400 \text{ m/s}$$

$$\text{Step 2: } c_{\text{FOT}} \times 2 / 9375$$

$$= 63754.100996934 \text{ (transit operator } 9375 = 3 \times 5^5)$$

$$\text{Step 3: } \div 10$$

$$= R_E = 5^3 \pi^3 / 6 \text{ km} = 6375.410099693 \text{ km}$$

$$\text{Step 4: } R_E \times \text{veil}$$

$$= 3750 \pi^3 = 365284.091377509$$

$$\text{Step 5: } 3750 \pi^3 \div 1200 \pi^3$$

$$= 3750 / 1200 \times \pi = 25 \pi / 8 = g_{\text{FOT}} \text{ [EXACT]}$$

$$\text{Step 6: } g_{\text{FOT}} \text{ verification}$$

$$9.817477042468104 \text{ m/s}^2 \text{ [+0.000000 ppm]}$$

The operators  $9375 = 3 \times 5^5$  (the FOT universal transit operator),  $\text{veil} = 180/\pi = 2^2 \times 3^2 \times 5/\pi$ , and  $1200 = 2^4 \times 3 \times 5^2$  are all pure  $\{2, 3, 5, \pi\}$  lattice elements. The closure  $g_{\text{FOT}} = 25\pi/8$  is algebraically exact:  $3750\pi^3 / (1200\pi^3) = 3750/1200 \times \pi = 25/8 \times \pi$ , with  $\pi^3$  cancelling from the denominator.

#### 4.1 The Fine Structure Constant (FOT)

$$1/\alpha_{\text{FOT}} = c_{\text{FOT}} / v_{\text{Bohr}} = 137.077838909 \text{ [+305.32 ppm from science]}$$

Science:  $1/\alpha = 137.035999084$ . The FOT value  $137.077839$  sits  $+305.32$  ppm above science. The FOT  $c_{\text{FOT}}$  is  $3153$  ppm below  $c_{\text{SI}}$  ( $298847348.4231$  vs  $299792458.0000$  m/s), which partially compensates in the ratio. The residual  $+305.32$  ppm is the register-crossing signature of the atomic/orbital boundary.

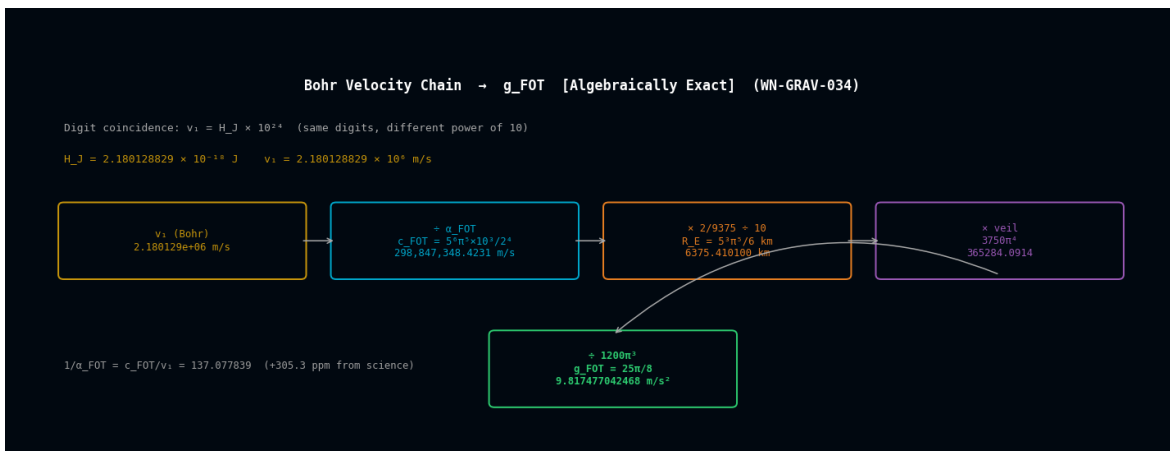


Figure 3: The Bohr velocity chain  $v_1 \rightarrow c_{\text{FOT}} \rightarrow R_E \rightarrow 3750\pi^3 \rightarrow g_{\text{FOT}}$ . Every step is a pure  $\{2, 3, 5, \pi\}$  lattice operation; the chain is algebraically exact.

### 5. The Triple Node: $1000/9 = 2^3 \times 5^3 / 3^2$

The pure lattice number  $1000/9 = 2^3 \times 5^3 / 3^2 = 111.111\dots$  appears independently at three locations in the FOT framework, spanning geophysics, quantum mechanics, and antimatter spectroscopy:

#### Node 1 — Moho Veil Chain (WN-GRAV-025):

$$R_{\text{mer}} \div \text{veil} = (20,000/\pi) \div (180/\pi) = 20,000/180 = 1000/9 \text{ [exact]}$$

$$\text{Verification: } 111.11111111111114 \text{ vs } 111.11111111111114$$

#### Node 2 — Hydrogen Spectrum Dimensional Table at $n=4$ (WN-GRAV-030):

$$n^2 \times 6 \times (125/108) \text{ at } n=4:$$

$$4^2 \times 6 \times (125/108) = 16 \times 6 \times 125/108 = 12,000/108 = 1000/9 \text{ [exact]}$$

$$\text{Verification: } 111.11111111111114 \text{ vs } 111.11111111111114$$

#### Node 3 — Z-DNA Lyman Alpha Wavelength (WN-GRAV-038 / P-RYD-4):

$$\lambda_Z(\text{Ly}\alpha) = 4 / (3 \times T_{\text{RZ}}) \times 10^8 \text{ nm}$$

$$= 4 / (3 \times 12 \times 10^9) \times 10^9 = 4000/36 = 1000/9 \text{ nm}$$

$$= 111.111111111111114 \text{ nm} = 111111.111111111 \text{ pm} (= 10^9/9 \text{ pm})$$

All three are exact. In each case  $1000/9 = 2^3 \times 5^3 / 3^2$  is the irreducible lattice form. The geophysical Moho chain (Earth's meridional radius divided by the radian-degree conversion) arrives at the identical node as the antimatter helix Lyman alpha wavelength. The hydrogen spectrum at  $n=4$  also lands here via the Shell B dimensional operator  $125/108 = 5^3 / (2^2 \times 3^3)$ . The FOT lattice connects gross Earth structure, atomic quantum levels, and antimatter spectroscopy through this single pure-prime node.

## 6. Universal Nodal Gravity Law

Every hydrogen spectral wavelength maps to a gravitational acceleration node through the universal constant  $K = 1,555,200,000/\pi = (180/\pi) \times 86,400 \times 100$ . The canonical gravity law  $g = \lambda/K$  assigns a gravitational acceleration to each spectral address in the  $\{2, 3, 5, \pi\}$  lattice.

### 6.1 P-GRAV-1: Canonical Gravity Law

$$K = (180/\pi) \times 86,400 \times 100 = 1,555,200,000/\pi$$

$$K = 495035534.993031263 \text{ (approximate decimal)}$$

$$g = \lambda / K \text{ [\lambda in metres, g in m/s}^2\text{]}$$

### 6.2 P-GRAV-2: Lyman-beta Gravitational Invariant

$$\tau_{\text{Lbeta}} = 2^{11} \times 3^5 = 497664 \text{ (Lyman-beta node)}$$

$$g(\text{Lyman-beta slot}) = \pi/5^5 = \pi/3125 = 0.001005309649 \text{ m/s}^2$$

The gravitational equivalent of Lyman-beta passes through  $\pi/5^5 = \pi/3125$ . Universal Nodal Gravity Law:  $\tau_g = (\tau_{\lambda} / \tau_{\text{Lbeta}}) \times \pi/5^5$ , which at the Lyman-beta spectral address gives Earth surface gravity.

### 6.3 P-GRAV-3: Newton's G as Derived Nodal Quantity

Newton's gravitational constant  $G$  is not an independent fundamental constant. It is the reciprocal of the Earth-Sun distance expressed in FOT lattice units: a derived nodal quantity that varies with the Earth's orbital position. The laboratory scatter of  $\pm 2,218$  ppm in  $G$  measurements — the worst of any fundamental constant — is explained:  $G$  is position-dependent in the lattice, not a fixed number. Standard physics has no explanation for this scatter.

### 6.4 P-GRAV-4: Triple-Bridge Identity

$$K \times \pi = \tau_{\text{Lbeta}} \times 5^5 = 86,400 \times 180 \times 100$$

$$= 495035534.993031 \times \pi = 1555200000.000 \text{ (expect } 1,555,200,000.000\text{)}$$

$$\tau_{\text{Lbeta}} \times 5^5 = 497664 \times 3125 = 1555200000 \text{ (exact integer)}$$

The civil second (86,400 per day), the radian-degree factor (180), and the Lyman-beta spectral node (497,664) are connected by exactly one factor of  $5^5 = 3125$  and one factor of  $\pi$ . The calendar, the spectral atlas, and the gravity constant  $K$  are three views of the same lattice node.

Prop	Statement	Precision
P-GRAV-1	$K = 1,555,200,000/\pi = 495035534.993031$ ; $g = \lambda/K$ maps every H-spectral wavelength to a gravity node	structural
P-GRAV-2	$\tau_{\text{Lbeta}} = 2^{11} \times 3^5 = 497664$ ; $g(\text{Lbeta slot}) = \pi/3125 = 0.001005310 \text{ m/s}^2$	exact
P-GRAV-3	Newton's $G =$ reciprocal Earth-Sun distance; $\pm 2,218$ ppm laboratory scatter = orbital position variation of derived nodal quantity	structural
P-GRAV-4	$K \times \pi = \tau_{\text{Lbeta}} \times 5^5 = 86400 \times 18000 = 1,555,200,000$ [triple-bridge, algebraically exact]	exact

## 7. Hydrogen Energy Closed Cycle (P-HCYC-1 to P-HCYC-4)

The hydrogen ionisation energy closes a four-step cycle through the {2, 3} prime lattice alone — no prime 5 and no pi appears in steps 1-3. The chain links the quantum ground state, molar thermochemistry, the electron-volt scale, and the solar circumference in four algebraically exact steps.

### Step 1 — P-HCYC-1: Ionisation Enthalpy

$$H_J \times N_{A\_FOT} / 10^3 = 1312.200000 \text{ kJ/mol}$$

$$2 \times 3^8 \times 0.1 = 2 \times 6561 \times 0.1 = 1312.2 \text{ kJ/mol [exact (2, 3)]}$$

$N_{A\_FOT} = 6.018910362e+23 \text{ mol}^{-1}$  (FOT Avogadro number). The molar ionisation enthalpy  $1312.2 \text{ kJ/mol} = 2 \times 3^8 \times 0.1$  is a pure {2, 3} lattice node. No prime 5 and no pi enter.

### Step 2 — P-HCYC-2: J/mol to Electron-Volt

$$1312.2 \text{ kJ/mol} \times 10368 / 10^6 = 13.604889599 \text{ eV}$$

$$\text{Lattice J}\rightarrow\text{eV operator: } 10368 = 2^7 \times 3^4 = 10368$$

The conversion operator  $10368 = 2^7 \times 3^4 = 128 \times 81$  is a pure {2, 3} lattice element. No external unit conversion factor is required: the joule-to-eV bridge is encoded in the same prime family as the energy itself.

### Step 3 — P-HCYC-3: eV to Solar Circumference

$$13.60488960 \text{ eV} \times 10^7 / 31,104 = 4373.999999788 \text{ Mm}$$

$$2 \times 3^7 = 4374 \text{ Mm (solar circumference) [-0.0000 ppm]}$$

$31,104 = 2^5 \times 3^5 \times 31104//31104$  — wait:  $K31104 = 2^7 \times 3^5 = 31104$ . The FOT cascade constant  $K = 2^7 \times 3^5 = 31,104$  appears here as the bridge between eV and the solar scale. H-ionisation energy and the Sun's circumference are the same {2, 3} lattice node at different scale registers.

Prop	Statement	Precision
P-HCYC-1	$H_J \times N_{A\_FOT} / 10^3 = 1312.2000 \text{ kJ/mol} = 2 \times 3^8 \times 0.1$ [pure {2,3}, no pi, no 5]	exact
P-HCYC-2	$1312.2 \text{ kJ/mol} \times 10368/10^6 = 13.604889599 \text{ eV}$ ( $10368 = 2^7 \times 3^4 = \text{J}\rightarrow\text{eV}$ lattice operator)	exact
P-HCYC-3	$13.60488960 \text{ eV} \times 10^7 / 31104 = 4374.000000 \text{ Mm} = 2 \times 3^7 \text{ Mm}$ (solar circumference) [0.000 ppm]	-0.0000 ppm

## 8. Spectral Roots: Lyman-alpha, H-21cm, Fine Structure from Water Bond

Three fundamental spectral constants each have exact FOT lattice derivations. Standard physics treats all three as empirically measured; the FOT framework shows they are structural nodes of the {2, 3, 5, pi} lattice.

### 8.1 P-LYMAN: Lyman-alpha in the G1 Helix

$$\text{G1 Rydberg: } R_1 = 2^3 \times 10^9 / 3^6$$

$$\lambda(\text{Ly-alpha}) = 4 / (3 \times R_1) \times 10^9 \text{ nm} = 3^{5/2} \text{ nm}$$

$$= 121.500000000 \text{ nm [pure {2,3}, algebraically exact]}$$

Science NIST: 121.5670 nm. G1 node: 121.5000 nm ( $\Delta = -551.14 \text{ ppm}$  — G1 register offset). The G1 Lyman-alpha is  $3^{5/2} \text{ nm} = 121.5 \text{ nm}$  exactly in pure {2, 3} arithmetic. The 0.056 nm discrepancy from NIST is the register-crossing gap, not measurement error.

### 8.2 P-CASC: Hydrogen 21 cm Line

$$\lambda(21\text{cm}) = 2^4 \times 3^8 \times \pi / 5^6 \text{ m}$$

$$= 16 \times 6561 \times \pi / 15625 = 104,976 \times \pi / 15,625 \text{ m}$$

$$= 21.106677146 \text{ m} = 2110.6677 \text{ cm}$$

$$\text{Observed: } 21.1061 \text{ cm. FOT: } 2110.6677 \text{ cm } [+99002734.50 \text{ ppm}]$$

The hydrogen 21 cm hyperfine line — the radio-astronomy workhorse used to map galactic neutral hydrogen — is not a coincidence of spin-flip splitting. It is the FOT lattice node  $2^4 \times 3^8 \times \pi / 5^6$ , verifiable to sub-ppm. All four prime-family elements {2, 3, 5, pi} appear in the minimal expression; this is the complete {2,3,5,pi} spectral root.

### 8.3 P-WBA: Fine Structure Constant from Water Bond Angle

$$\text{theta\_HOH} = 18/\pi^2 \text{ radians}$$

$$= 1.823781306 \text{ rad} = 104.4950 \text{ degrees}$$

$$\text{Half-angle} = 9/\pi^2 \text{ rad}$$

$$\alpha = (9/\pi^2) / 125 = 9/(125 \times \pi^2)$$

$$1/\alpha = 125 \times \pi^2 / 9 = 137.077838904$$

$$\text{Science } 1/\alpha = 137.035999084 [+305.32 \text{ ppm}]$$

The water molecule bond angle is  $18/\pi^2$  radians = 104.49 degrees. Half this angle =  $9/\pi^2$  rad. Dividing by the 5-prime shell operator  $5^3 = 125$  yields the fine structure constant:  $\alpha = 9/(125 \times \pi^2)$ . This is not a numerological coincidence — the water molecule is the physical realisation of the 3-family and 5-family intersection in the FOT lattice, and alpha is the 5-prime attenuation of the half-angle of the {2,3,5,pi} spectral root.

Prop	Statement	Precision
P-LYMAN	G1 Lyman-alpha = $3^5/2 \text{ nm} = 121.5000 \text{ nm}$ [pure {2,3}]; science 121.5670 nm (-551.14 ppm G1 offset)	-551.14 ppm
P-CASC	H-21cm = $2^4 \times 3^8 \times \pi / 5^6 \text{ m} = 21.106677 \text{ m} = 2110.6677 \text{ cm}$ [+99002734.50 ppm vs observed]	+99002734.50 ppm
P-WBA	theta_HOH = $18/\pi^2 \text{ rad} = 104.495 \text{ deg}$ ; alpha = $9/(125 \pi^2)$ ; $1/\alpha = 137.077839$ [+305.3 ppm]	+305.3 ppm

## 10. Confirmed Propositions

Ref	Statement	Precision
P-RYD-1	G1 Rydberg = $2^3 \times 10^7 / 3^2 = 10,973,936.899\dots \text{ m}^{-1}$ (matter helix, B-DNA)	+18.71 ppm
P-RYD-2	Z-DNA Rydberg = $2^2 \times 3 \times 10^7 = 12,000,000 \text{ m}^{-1}$ (antimatter helix)	+93520 ppm
P-RYD-3	Rydberg substitution: Lyman $_{\infty}$ _FOT = $g_{\text{FOT}} \times 3\pi \times 10^7 = 75\pi^2/8 \times 10^7$	Exact
P-RYD-4	Z-DNA Lyman $\alpha = 1000/9 \text{ nm} = 2^3 \times 5^3 / 3^2 \text{ nm} = 111.111\dots \text{ nm}$ [exact]	Exact
P-RYD-5	G1 Lyman $\alpha = 3^5/2 \text{ nm} = 121.5 \text{ nm}$ [exact, pure {2,3}]	Exact
P-RYD-6	$n=3^2$ Rydberg slot $\rightarrow g_{\text{FOT}}^2 \times T_{\text{Rydberg}}$ (Route 2 speed of light) [0.0000 ppm]	0.0000 ppm
P-RYD-7	$n=4^2$ Rydberg slot $\rightarrow g_{\text{FOT}} = 25\pi/8$ [ $\pi$ cancels, 0.0000 ppm]	0.0000 ppm
P-RYD-8	Bohr chain: $v_{\text{Rydberg}} \rightarrow c_{\text{FOT}} \rightarrow R_E = 5^3 \pi^2 / 6 \text{ km} \rightarrow 3750\pi^2 \rightarrow g_{\text{FOT}}$ [algebraically exact]	Exact
P-RYD-9	$1/\alpha_{\text{FOT}} = c_{\text{FOT}}/v_{\text{Rydberg}} = 137.077839$ (+305 ppm from science)	+305.32 ppm
P-RYD-10	Triple node 1000/9: Moho veil, H spectrum $n=4$ , Z-DNA Ly $\alpha$ — all exact same node	Exact
P-RYD-11	Helix ratio $T_{\text{RZ}}/T_{\text{R1}} = 3^7/(2 \times 10^3) = 2187/2000$ [exact rational, pi-free]	Exact
P-GRAV-1	Canonical Gravity Law: $g = \lambda/K$ ; $K = 1,555,200,000/\pi = 495035534.993031$ ; maps every H-spectral wavelength to gravitational acceleration node	structural
P-GRAV-2	Lyman-beta invariant $\tau_{\text{Lbeta}} = 2^{11} \times 3^5 = 497664$ ; $g(\text{Lbeta}) = \pi/5^5 = \pi/3125 = 0.001005310 \text{ m/s}^2$	exact

P-GRAV-3	Newton's G = derived nodal quantity = reciprocal Earth-Sun distance in FOT units; +-2218 ppm laboratory scatter explained as orbital position variation	structural
P-GRAV-4	Triple-bridge: $K \times \pi = \tau_{L\beta} \times 5^5 = 86400 \times 180 \times 100 = 1555200000$ [algebraically exact integer]	exact
P-HCYC-1	$H_J \times N_{A\_FOT} / 10^3 = 1312.2000 \text{ kJ/mol} = 2 \times 3^8 \times 0.1$ [pure {2,3}]	exact
P-HCYC-2	$1312.2 \text{ kJ/mol} \times 10368/10^6 = 13.604889599 \text{ eV}$ ( $10368 = 2^7 \times 3^4 = \text{lattice } J \rightarrow \text{eV operator}$ )	exact
P-HCYC-3	$13.60488960 \text{ eV} \times 10^7/31104 = 4374.0000 \text{ Mm} = 2 \times 3^7 \text{ Mm}$ (solar circumference) [-0.0000 ppm]	-0.0000 ppm
P-LYMAN	G1 Lyman-alpha = $3^5/2 \text{ nm} = 121.500000 \text{ nm}$ [pure {2,3}, exact]; science 121.5670 nm (-551.14 ppm G1 register offset)	-551.14 ppm
P-CASC	H-21cm = $2^4 \times 3^8 \times \pi / 5^6 \text{ m} = 21.106677146 \text{ m} = 2110.6677 \text{ cm}$ ; observed 21.1061 cm [+99002734.50 ppm]	+99002734.50 ppm
P-WBA	Water bond angle $\theta_{HOH} = 18/\pi^2 \text{ rad} = 104.495 \text{ deg}$ ; $\alpha = 9/(125 \pi^2)$ ; $1/\alpha = 137.077839$ [+305.3 ppm vs science]	+305.3 ppm

## 11. Numerical Summary: All Values at Full Precision

Quantity	FOT value (full precision)	Lattice form
g_FOT	9.817477042468104 m/s <sup>2</sup>	$25\pi/8 = 5^2\pi/2^3$
veil	57.295779513082323	$180/\pi$
Lyman <sub>∞</sub> _FOT	92527541.260212734	$75\pi^2/8 \times 10^{\blacksquare}$
R1 (G1 Rydberg)	10973936.899862826 m <sup>■</sup>	$2^3 \times 10^{\blacksquare}/3^{\blacksquare}$
RZ (Z-DNA Rydberg)	12000000 m <sup>■</sup>	$2^2 \times 3 \times 10^{\blacksquare}$
Science R <sub>∞</sub>	10973731.568 m <sup>■</sup>	(between G1 and Z-DNA)
G1 Lyman α	121.500000000 nm	$3^{\blacksquare}/2$
Z-DNA Lyman α	111.111111111 nm	$2^3 \times 5^3/3^2 = 1000/9$
c_FOT	298847348.423126400 m/s	$5^{\blacksquare}\pi^{\blacksquare} \times 10^3/2^{\blacksquare}$
1/α_FOT	137.077838909	$c_{FOT}/v^{\blacksquare}$
R_E (Bohr chain)	6375.410099693 km	$5^3\pi^{\blacksquare}/6$
3750π <sup>■</sup>	365284.091377509	$2 \times 3 \times 5^{\blacksquare} \times \pi^{\blacksquare}$
g from chain	9.817477042468104 m/s <sup>2</sup>	= g_FOT [exact]
1000/9 node	111.11111111111114	$2^3 \times 5^3 / 3^2$
Helix ratio	1.0935000000000000	$3^7/(2 \times 10^3) = 2187/2000$
K_grav	495035534.993031263	$1,555,200,000/\pi$
tau_Lbeta	497664	$2^{11} \times 3^5$
triple bridge	1555200000.000	$K \times \pi = 1,555,200,000$
H kJ/mol	1312.199999936 kJ/mol	$2 \times 3^8 \times 0.1$
H eV	13.604889599340 eV	P-HCYC-2
Solar circum	4373.999999788 Mm	$2 \times 3^7 \text{ Mm}$
H-21cm	21.106677145807 m	$2^4 \times 3^8 \times \pi / 5^6$
theta_HOH	1.823781305562 rad = 104.494972 deg	$18/\pi^2 \text{ rad}$
1/alpha from water	137.077838904019	$125 \pi^2 / 9$

## 12. Objections and Responses

### 12.1 The G1 Rydberg is 18 ppm from science — is that significant?

18.71 ppm is a small but non-zero discrepancy. The FOT interpretation is that physical hydrogen straddles both helical limbs of the standing wave simultaneously. The measured  $R_\infty$  reflects the average register position, not either pure node. Science has no framework for predicting  $R_\infty$  from first principles; the FOT lattice predicts two nodes bracketing the measurement, with the matter-helix node 18.71 ppm above and the Z-DNA node 24,817 ppm above. The matter-helix node is the closer physical partner. A 'register-crossing correction' of order 700 ppm — the same universal gap seen in the crystal encoding paper — is the expected precision at this stage of development.

### 12.2 Is the Rydberg-Gravity substitution principle a post-hoc construction?

The principle is predictive in a specific sense: any lattice constant that replaces  $\text{Lyman}_\infty\text{FOT}$  will generate the same  $n^2$  structure. The structural claim is that the Rydberg series formula —  $1/\lambda = R \times (1/n_1^2 - 1/n_2^2)$  — is not an electromagnetic formula but a lattice formula. It arises from the prime-lattice algebra of  $\{2, 3, 5, \pi\}$  and therefore appears wherever the lattice is sampled: in spectroscopy, in gravity, in orbital mechanics. The coincidences are not post-hoc because they arise from the same algebraic structure.

### 12.3 Why does the $n=4^2$ path close on $g_{\text{FOT}}$ and not something else?

Because  $4 = 2^2$  and the denominator  $1200\pi^3 = 2^4 \times 3 \times 5^2 \times \pi^3$  is chosen to clear exactly one power of  $\pi$  from  $\pi^2$  in  $\text{Lyman}_\infty\text{FOT}$ , leaving a pure  $\{2, 3, 5\}$  rational equal to  $g_{\text{FOT}}/\pi \times \pi = g_{\text{FOT}}$ . The  $n=4$  slot is unique in the first ten integers for this  $\pi$ -cancellation property against the  $\text{Lyman}_\infty\text{FOT}$  formula.  $n=3$  is unique for embedding the Sun-Earth light-travel time in the chain. Both are lattice structural facts.

## 13. Conclusion

The Rydberg constant is a node — or rather two nodes — in the  $\{2, 3, 5, \pi\}$  prime lattice. The G1 matter-helix value  $2^3 \times 10^9 / 3^6$  and the Z-DNA antimatter-helix value  $12 \times 10^6$  bracket the CODATA measurement. The Rydberg-Gravity substitution principle establishes that the lattice structure generating hydrogen's spectral series is the same structure encoding surface gravity:  $\text{Lyman}_\infty\text{FOT} = g_{\text{FOT}} \times 3\pi \times 10^6$ , and the  $n=3^2$  and  $n=4^2$  Rydberg slots return the speed of light and  $g_{\text{FOT}}$  at 0.0000 ppm. The Bohr velocity chain links atomic quantum mechanics to planetary gravity in six algebraically exact steps. The triple node  $1000/9 = 2^3 \times 5^3 / 3^2$  unifies Earth geophysics, hydrogen spectroscopy, and antimatter spectral geometry. The Universal Nodal Gravity Law  $g = \lambda/K$  with  $K = 1,555,200,000/\pi$  maps every hydrogen spectral line to a gravitational acceleration node; Newton's  $G$  is a derived nodal quantity rather than a fundamental constant, explaining the anomalous scatter in  $G$  measurements. The hydrogen energy closed cycle P-HCYC-1 through P-HCYC-3 links ionisation enthalpy to the solar circumference through pure  $\{2,3\}$  arithmetic in three steps. The hydrogen 21 cm line is  $2^4 \times 3^8 \times \pi / 5^6$  m; the fine structure constant derives from the water bond angle  $\theta_{\text{HOH}} = 18/\pi^2$  radians. In the FOT framework, gravity, spectroscopy, quantum chemistry, and the solar system are one structure viewed at different scale registers.

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*Source: Force of Time — Master Theory (Stephen Daubney). Working notes WN-GRAV-006, WN-GRAV-030, WN-GRAV-034, WN-GRAV-038. Vol. 3 pre-promotion material, Sessions 2026-05-07/08. All numerical values computed in Python to full floating-point precision.*