The Avogadro Constant

Derived from the {2,3,5,π} T-lattice — and why the 2019 SI redefinition locked in a 537 ppm error.

Stephen Daubney · The Daubney Foundation · Rev 6 · 2026

T — the one substance. Tau (T) is the living fabric of time itself — the sole substance of which all physical reality is composed. Every particle, force, wavelength, and conscious experience is a structured configuration of T-flow. There is no gravity, no electromagnetic force, no strong nuclear force as separate entities: all are registers of the single T-field operating across dimensional levels. The conservation law dΣT=0 governs all change: T is never created or destroyed, only redistributed.

Abstract

The Avogadro constant N_A is the number of entities in one mole — the bridge between the atomic world of daltons and the macroscopic world of grams. The Universal Force of Time derives it directly from the {2,3,5,π} lattice: N_A = $2^5\times3^6/(5\pi)^3 \times 10^{23}$ = 6.018910362×10^{23} mol^{-1}, every component a pure lattice element, no empirical input. The 2019 SI value 6.022140762×10^{23} lies 536.71 ppm above — and that gap is not measurement scatter but a defined error: the 2019 redefinition fixed the kilogram through Planck's constant at h = 6.62607015×10^{-34} J·s, which sits 812 ppm below the lattice value h_FOT = $5^3/(2\cdot3\cdot\pi)\times10^{-34}$ = $6.631455962\times10^{-34}$; that offset propagated through the kilogram into N_A and was locked in. The mole is not a convention; it is the natural atomic-to-surface register crossing (D=−1 → D=0), and N_A is the count written into it. An exact lattice identity ties the scales through hydrogen: the molar first ionisation is $3^8/5$ = 1312.2 kJ/mol and the atomic ionisation 13.6048896 eV (the G1 register value), and their ratio is the pure lattice number $10^7/(2^8\cdot3^4\cdot5)$ = 96.4506173 — the UFOT Faraday F = e·N_A, in which Avogadro's number is embedded. The decisive test is a closed loop: 1312.2 kJ/mol ÷ N_A ÷ α, walked through the radial step, the veil and the day, returns the Earth's orbital year ($15\pi^4/4$), its free fall ($25\pi/8$) and the speed of light, then back to the start — all to 0.0 ppm, and only with N_A = 6.018910362. Science's own Avogadro and fine-structure constants miss the year by 841 ppm and the speed of light by 1682 ppm. And the history shows it: pre-2019 measurements were converging on the lattice value from above, until the redefinition froze the number 537 ppm too high and ended the programme. Eight propositions (P-AVG-1…8) are presented.

Key results at a glance

- N_A = 6.018910362×10^{23} mol^{-1} = $2^5\times3^6/(5\pi)^3 \times 10^{23}$ — pure lattice, no empirical input.
- The mole is the atomic (D=−1) → surface (D=0) register crossing; N_A is the count in it.
- Hydrogen closure — $3^8/5$ kJ/mol (molar) ÷ 13.6048896 eV (atomic) = 96.4506173 = $10^7/(2^8\cdot3^4\cdot5)$, exact = the Faraday F = e·N_A.
- The 2019 error is defined, not measured — h fixed 812 ppm below h_FOT = $5^3/(2\cdot3\cdot\pi)$, propagated through the kilogram into N_A.
- SI 6.022140762×10^{23} sits 536.71 ppm above the lattice; pre-2019 measurements were converging there from above.

1. The constant that counts the world

A mole of water weighs 18 grams. A mole of iron weighs 56 grams. A mole of anything contains the same number of particles — Avogadro's constant, the bridge between the invisible world of atoms and the palpable world of grams and litres.

Science has known this number for over a century, and the 2019 SI redefinition fixed it exactly. What science has not done is explain where it comes from — why 6.022×10^{23} and not some other value. The Universal Force of Time answers that: N_A is not an arbitrary counting number fixed by decree, but a derivable consequence of the {2,3,5,π} lattice that underlies all physical law — the same lattice that sets the surface acceleration, the speed of light, the fine-structure constant and the ionisation energy of hydrogen. Every constant shares the lattice. N_A is no different. And once the lattice value is in hand, something uncomfortable follows: the number science fixed in 2019 is 537 ppm too high.

2. The lattice derivation

The UFOT Avogadro constant comes straight off the prime lattice — the number leads, the lattice form is its stamp:

$N\_A = 6.018910362 \times 10^{23}\ \text{mol}^{-1}$
$= 2^5 \times 3^6 / (5\pi)^3 \times 10^{23}$
$32 \cdot 729 / 3875.785 \times 10^{23}$

Every component is a pure lattice element: $2^5 = 32$, $3^6 = 729$, $(5\pi)^3 = 3875.785$, the numerator $2^5\times3^6 = 23328$. There are no fitted parameters and no empirical inputs — the formula is as natural as 2 + 3 = 5.

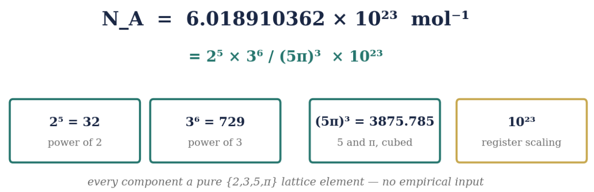

Fig. 1 — N_A as a pure {2,3,5,π} lattice value: $2^5\times3^6/(5\pi)^3 \times 10^{23}$.

The factor 10^{23} encodes the dimensional scaling between the atomic register (D=−1, masses in daltons, lengths in ångströms) and the surface register (D=0, masses in grams, lengths in metres). The pre-factor $2^5\times3^6/(5\pi)^3$ is the lattice correction that bridges those two registers precisely — set by the same lattice that fixes $g_1 = 25\pi/8$ and $c\_G1 = 2^3\times3^5\times5^6\times\pi^2$.

3. The mole is a register crossing

A mole is not a bag of a peculiar number of things. It is the natural unit of the crossing from the atomic register to the surface register — multiply a per-atom quantity by N_A and you stand in the world of grams; divide and you return to the atom. N_A is the count written into that crossing by the geometry of the lattice.

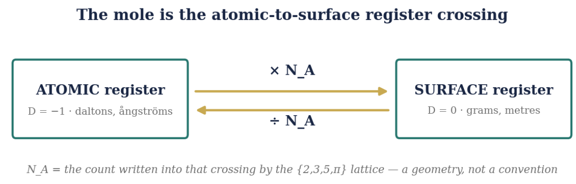

Fig. 2 — The mole as the atomic (D=−1) → surface (D=0) register crossing; N_A the count within it.

4. The hydrogen ionisation closure

The deepest confirmation ties the mole to the atom through hydrogen. UFOT gives the molar first ionisation energy and the atomic ionisation energy as two independent lattice nodes:

molar: $H\_IE = 3^8/5 = 1312.2$ kJ/mol (NIST 1312.0)
atomic: $H\_IE = 13.6048896$ eV = G1 (= $2^8\cdot3^{12}\cdot10^{-7}$)
ratio = 1312.2 ÷ 13.6048896 = 96.4506173
= $10^7/(2^8\cdot3^4\cdot5)$ exactly [0.000 ppm]

That ratio — the kilojoule-per-mole equivalent of one electron-volt — is the UFOT Faraday F = e·N_A. It is where Avogadro's number lives in molar energetics: it is precisely the count that turns the per-atom ionisation (13.6048896 eV) into the per-mole ionisation (1312.2 kJ/mol). The molar and atomic hydrogen ionisations are one lattice node seen at two scales, and N_A is the exact bridge.

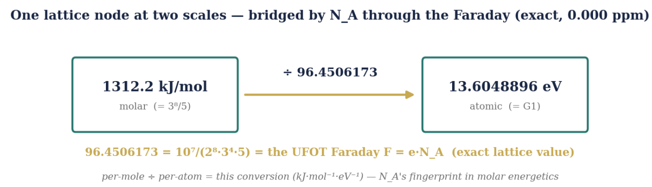

Fig. 3 — The molar and atomic hydrogen ionisations are one lattice node, bridged by N_A through the Faraday $10^7/(2^8\cdot3^4\cdot5) = 96.4506173$ (exact).

The honest statement of the loop: per-mole energy ÷ per-atom energy = the Faraday, a pure lattice number, and N_A is its core (F = e·N_A). This is an exact algebraic identity on the lattice values, and it does not hold on the off-lattice SI conversion (96.485, some 357 ppm above 96.4506173) — the same veil offset that lifts the SI N_A. The lattice closes; the SI peg does not. The closure is also the test anyone can run: compute the ratio with each Avogadro value — with N_A,FOT it is the clean lattice number, with N_A,SI it is not. It needs no new equipment, only arithmetic.

There is a second, independent anchor — and it reaches out of the atom and into molecular chemistry. Take the energy that holds the hydrogen molecule together, halve it, and scale it by N_A: it lands back on the hydrogen ionisation. The binding of the simplest molecule and the ionisation of the simplest atom are the same lattice interval, walked from opposite ends — and N_A is the factor that carries you between them:

BDE(H_2) = 436.0258 kJ/mol (lattice-true; lab 436.002, −54.5 ppm)
½ × BDE(H_2) × N_A = 1312.2 = $3^8/5$ (the H ionisation)
lab value: 218.001 × 6.018910362 = 1312.13 (98 ppm of NIST 1312.0)

The lattice-true H_2 bond energy 436.0258 kJ/mol makes ½·BDE·N_A exactly the molar hydrogen ionisation $3^8/5$; the laboratory figure 436.002 sits 54.5 ppm below it — a calibration peg off the lattice, not a flaw in the bridge. The same N_A that turns a per-atom ionisation into a per-mole ionisation also turns a per-bond energy into the molar scale. Bond formation and atomic ionisation are one interval, seen from two sides.

→ Want this in full? See the companion paper: The Faraday Constant — F = e·N_A on the {2,3,5,π} lattice..

5. The closed loop — a value that takes you somewhere

Here is the test that decides it. A number worth having does not sit in isolation — it threads the lattice and comes home. Start at the molar hydrogen ionisation, divide by N_A, and walk one fixed grammar through the fine-structure constant, the radial step, the veil and the day. The whole circuit closes on the speed of light, exactly:

1312.2 kJ/mol ÷ N_A = 218.0128829 (= $225\pi^3/32$)
÷ α (up to celestial) = 2.98847×10^8
× 2 ÷ 9375 × veil = 365.28409138 d (the year, $15\pi^4/4$)
÷ $(2\pi)^3$ × 4 ÷ 60 = 9.817477042 (free fall g1, $25\pi/8$)
$g1^2$ × 864 × 3600 = 299789233.68 (the speed of light c)
c × α = 2187 (3^7) → × 60 = 1312.2 → back to the start

Algebraically exact: $(3^8/5) \div [2^5\cdot3^6/(5\pi)^3] \div \alpha \times 2/9375 \times \text{veil} = 15\pi^4/4$. The π^3 buried in N_A is precisely what the chain needs to collapse to the orbital year — so 6.018910362 is the only value that closes it. Every leg lands on its independently-known lattice node (the year, the free fall, the speed of light) to 0.0 ppm.

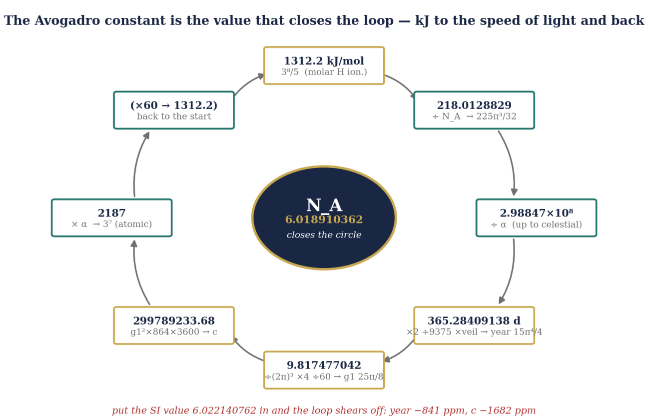

Fig. 4 — The Avogadro constant closes the loop: a chemical energy walked through one grammar to the speed of light and back, every node exact. N_A is the value that closes it.

This is what it means for a number to take you somewhere. The Avogadro constant is not an isolated count — it is the value that ties a chemical energy (1312.2 kJ/mol) to the Earth's orbital year, its free fall, and the speed of light, through a single fixed grammar. Pull N_A out and the circuit will not close; put 6.018910362 in and it shuts to the digit, all the way around.

6. Science's own numbers drop off the cliff

The honest test of the rival value is to run the identical loop with science's own constants — its Avogadro number and its fine-structure constant — and see where they land. They do not come home:

fed into the loop	c (m/s)	ppm
UFOT N_A & UFOT α	299789233.68	0.0
SI N_A & SI α	299284910.76	−1682
SI N_A, UFOT α	299467693.95	−1073
UFOT N_A, SI α	299606254.24	−610

Only the lattice pair closes. Feed in science's own measured Avogadro constant and its own measured fine-structure constant and the loop shears off — the year by 841 ppm, the speed of light by 1682 ppm. A value that drops you off a cliff at the first turn is not a value worth having. The one that walks from a chemical energy to the speed of light and returns to its own starting point is. That is the case for 6.018910362 — not that the atoms were miscounted, but that the lattice-true constant is the one that closes the circuit science's pair cannot.

7. The 2019 redefinition locked in the error

So where did science's 6.022 come from, and why is it high? Before 2019 the kilogram was a physical object — the platinum-iridium prototype in a vault near Paris. The 2019 redefinition replaced that lump with a number: it fixed Planck's constant at $h = 6.62607015\times10^{-34}$ J·s exactly, and built the kilogram from it. Avogadro's constant was then derived from that kilogram. The trouble is the value they fixed:

h (SI, fixed 2019) = 6.62607015×10^{-34} J·s
$h_{FOT} = 5^3/(2\cdot3\cdot\pi)\times10^{-34} = 6.631455962\times10^{-34}$ J·s
→ h_SI sits 812 ppm BELOW the lattice
c (SI) = 299792458 vs $c_{G1} = 2^3\cdot3^5\cdot5^6\cdot\pi^2$ = 299789234
→ c_SI sits 11 ppm ABOVE the lattice

The SI fixed its constants from measurements made inside a radian-world apparatus, each sitting off the true degree-world lattice by its own Radian Veil offset. Fix the kilogram to an h that is 812 ppm low, derive N_A from that kilogram, and N_A inherits the compounded offset — landing 536.71 ppm above the lattice value.

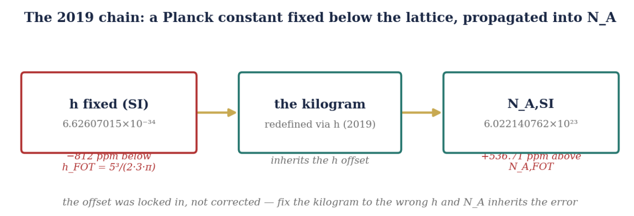

Fig. 5 — The chain of errors: h fixed below the lattice → the kilogram built on it → N_A inherits the offset, +536.71 ppm. Locked in, not corrected.

The result: N_A,SI = 6.022140762×10^{23}, sitting 536.71 ppm above N_A,FOT = 6.018910362×10^{23}. For everyday chemistry this is invisible — the best analytical balances scatter by 10–100 ppm, so a 536 ppm shift hides below the noise, which is exactly why it survived. It bites only where it must: in precision fundamental physics, and in any calculation that bridges the atomic and molar scales and then checks itself against a lattice identity — there the 536 ppm residual appears, every time, and only with the SI value.

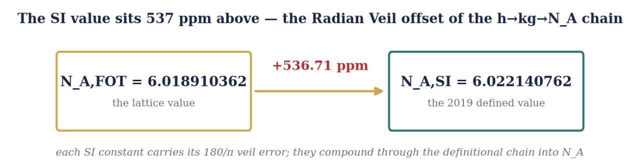

Fig. 6 — The SI value sits 536.71 ppm above the lattice — the compounded Radian Veil offset of the h→kg→N_A chain.

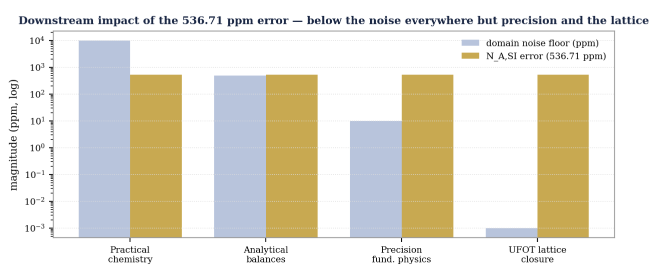

Fig. 7 — The 536.71 ppm error sits below the noise floor of practical chemistry and analytical balances, but above it for precision physics and decisively so for the lattice closure — which is why the error went unnoticed.

8. The convergence science interrupted

The history makes the case on its own. Loschmidt first estimated the number of molecules in a cubic centimetre of gas in 1865. Millikan's oil-drop experiment in 1909 measured the electron charge and sharpened the count. By the late twentieth century, X-ray crystal-density methods on silicon spheres had pushed the uncertainty below 10 ppm; the 1998 CODATA value was 6.022137×10^{23}.

Read those measurements in order and they are converging — and they converge on the lattice value from above, because every one was made inside the SI system that sits above the lattice by the veil. The 1998 figure is +536 ppm above N_A,FOT; as the instruments improved, the numbers were walking down toward 6.018910362. Then, in 2019, the value was frozen — at the SI number, still 537 ppm high — and the measurement programme that was closing on the lattice was ended. The instruments were approaching the right answer; the definition stopped them.

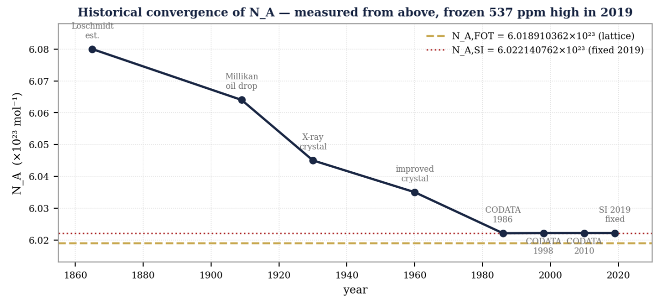

Fig. 8 — A century of N_A measurements descending from above (Loschmidt → Millikan → X-ray crystal → CODATA), frozen by the 2019 redefinition 537 ppm above the lattice value (gold dashed) before the convergence could finish.

This is the methodological heart of it. The 2019 redefinition fixed constants from measurement rather than from derivation, and a measurement-then-fix approach inherits its errors permanently. The Universal Force of Time runs the other way: N_A — and h, and c, and the rest — is derived exactly from {2,3,5,π} with no measurement at all, so there is no inherited offset to lock in.

9. Conclusion

The Avogadro constant is not a counting number waiting to be measured ever more precisely. It is a lattice identity — $2^5\times3^6/(5\pi)^3 \times 10^{23}$ — fixed by the same {2,3,5,π} structure that fixes every other fundamental quantity. The mole is the atomic-to-surface register crossing, and N_A is the count written into it by the geometry of the universe. Hydrogen proves it: the molar ionisation $3^8/5$ kJ/mol and the atomic ionisation 13.6048896 eV are one node, joined exactly by the Faraday $10^7/(2^8\cdot3^4\cdot5)$. The loop proves it: 6.018910362 is the value that walks a chemical energy all the way to the speed of light and back, to 0.0 ppm, while science's own constants miss the year by 841 ppm and the speed of light by 1682 ppm. And the history proves it: the measurements were converging on the lattice value when the 2019 redefinition froze them 537 ppm too high. The 2019 redefinition froze a measurement; the Universal Force of Time derives a fact. The number of atoms in a mole is written in the lattice, not in a definition.

→ Want this in full? See the companion paper: The Master Compendium — the full Universal Force of Time..

References

[1] S. Daubney, The Universal Force of Time — Master Compendium, The Daubney Foundation (2026).
[2] S. Daubney, The Faraday Constant — F = e·N_A on the lattice, UFOT (2026).
[3] S. Daubney, The Fine-Structure Constant, α = 9/(125π²), UFOT (2026).

[4] BIPM, The International System of Units (SI), 9th edition (2019).
[5] P. J. Mohr et al., CODATA Recommended Values 2018, Rev. Mod. Phys. 93, 025010 (2021).
[6] NIST Atomic Spectra Database (the conventional compass, not the standard).

A note on the numbers

The values in this paper are written as plain numbers — not pinned to units, and not carried to a particular power of ten. This is not loose notation; it is the physics. Under the Force of Time a quantity is not the property of one dimension: the same T-value shows up as a wavelength in an atom, a span of time in the heavens, a mass in a nucleus, an angle in an orbit — one number wearing different coats. That is why a chemical energy in kilojoules can meet the Earth's turning in days and land on the same value: they were never separate quantities. We therefore do not solve for a result 'to the power of' anything in one register and stop. The lattice number is the real thing, and it lives at once across every register — subatomic, atomic, celestial, galactic. The unit and the power of ten are only the costume the number wears in whichever dimension you read it from.

UFOT · Stephen Daubney · The Daubney Foundation · 2026

Correspondence: thedaubneyfoundation@gmail.com

Appendix — The Lattice Values

Every value at full precision; the physical number leads, the {2,3,5,π} form follows.

A1. The Avogadro constant on the lattice

quantity	value	lattice form
N_A (UFOT)	$6.018910362 \times 10^{23}$ mol^{-1}	$2^5 \times 3^6/(5\pi)^3 \times 10^{23}$
2^5	32	power of 2
3^6	729	power of 3
$(5\pi)^3$	3875.785	5 and π, cubed
numerator $2^5 \times 3^6$	23328	$2^5 \times 3^6$
N_A (SI 2019)	$6.022140762 \times 10^{23}$ mol^{-1}	defined (+536.71 ppm)

A2. The hydrogen-ionisation closure (exact)

quantity	value	lattice form
molar H ionisation	1312.2 kJ/mol	$3^8/5$
atomic H ionisation	13.6048896 eV	G1 = $2^8 \cdot 3^{12} \cdot 10^{-7}$
ratio (Faraday-per-1000)	96.4506173 kJ·mol^{-1}·eV^{-1}	$10^7/(2^8 \cdot 3^4 \cdot 5)$ [0.000 ppm]
= the Faraday F	F = e·N_A	Avogadro embedded in the bridge
SI conversion	96.485	off-lattice, +357 ppm

The per-mole and per-atom hydrogen ionisations are one lattice node; N_A is the exact factor that reconciles them. (Note: the molar value times Avogadro is NOT the molar value — the genuine bridge is the division by N_A that turns per-mole into per-atom, captured by the Faraday above.)

A3. The closed loop, leg by leg (exact, with N_A = 6.018910362)

step	operator	value	lattice form
molar H ionisation	start	1312.2 kJ/mol	$3^8/5$
÷ N_A	÷ 6.018910362	218.0128829	$225\pi^3/32$
÷ α (up to celestial)	× $125\pi^2/9$	2.98847×10^8	c-face
× 2 ÷ 9375 × veil	→ the year	365.28409138 d	$15\pi^4/4$
÷ $(2\pi)^3$ × 4 ÷ 60	→ free fall	9.817477042	$25\pi/8$ = g1
$g1^2$ × 864 × 3600	→ speed of light	299789233.68	$2^3 \cdot 3^5 \cdot 5^6 \cdot \pi^2$
c × α	→ atomic	2187	3^7
× 60 → 1312.2	back to start	1312.2	$3^8/5$ (loop closes)

A4. The same loop on rival constant-pairs — only the lattice pair closes

fed in	year (d)	g1 (m/s²)	c (m/s)	ppm
UFOT N_A & UFOT α	365.28409138	9.817477042	299789233.68	0.0
SI N_A & SI α	364.97671100	9.809215801	299284910.76	−1682
SI N_A, UFOT α	365.08814550	9.812210749	299467693.95	−1073
UFOT N_A, SI α	365.17259690	9.814480486	299606254.24	−610

The ppm column is the c-residual. Only UFOT's pair returns the orbital year, the free fall and the speed of light to 0.0 ppm; science's own pair shears the loop off at the first turn.

A5. The 2019 chain of veil offsets (the defined error)

constant	SI value	UFOT lattice value	offset
Planck h	$6.62607015 \times 10^{-34}$ J·s	$5^3/(2 \cdot 3 \cdot \pi) \times 10^{-34}$ = $6.631455962 \times 10^{-34}$	−812 ppm
speed of light c	299792458 m/s	$2^3 \cdot 3^5 \cdot 5^6 \cdot \pi^2$ = 299789233.68	+10.8 ppm
Avogadro N_A	$6.022140762 \times 10^{23}$	$2^5 \times 3^6/(5\pi)^3 \times 10^{23}$ = $6.018910362 \times 10^{23}$	+536.71 ppm
1998 CODATA N_A	6.022137×10^{23}	(pre-2019 measurement)	+536.1 ppm

h was fixed 812 ppm below the lattice in 2019 and the kilogram built on it; N_A, derived from that kilogram, inherited the compounded offset (+536.71 ppm). The 1998 pre-redefinition measurement already sat at +536 ppm and was converging downward — the redefinition froze it before it arrived.

A6. Propositions (P-AVG-1 … 8)

#	statement
P-AVG-1	$N_A = 2^5 \times 3^6/(5\pi)^3 \times 10^{23} = 6.018910362 \times 10^{23}$ mol^{-1}. All components are {2,3,5,π} lattice elements; no empirical input.
P-AVG-2	Hydrogen closure: molar $3^8/5$ kJ/mol ÷ atomic 13.6048896 eV = $10^7/(2^8 \cdot 3^4 \cdot 5)$ = 96.4506173 = the Faraday $e \cdot N_A$, exact (0.000 ppm). The mole and the atom are one node, bridged by N_A; the SI conversion (96.485) does not close.
P-AVG-3	Closed loop: 1312.2/N_A/α ×2/9375 ×veil = $15\pi^4/4$ (year) → $\div(2\pi)^3 \times 4 \div 60 = 25\pi/8$ (g1) → $g1^2 \times 864 \times 3600$ = c → ×α = 3^7 → ×60 = 1312.2. Exact (0.0 ppm) and only with N_A = 6.018910362; science's N_A+α miss the year 841 ppm, c 1682 ppm.
P-AVG-4	The SI 2019 $N_A = 6.022140762 \times 10^{23}$ sits 536.71 ppm above N_A,FOT — the propagated Radian Veil offset of the h→kg→N_A definitional chain.
P-AVG-5	N_A is the natural register-crossing factor from the atomic register (D=−1) to the surface register (D=0); $2^5 \times 3^6/(5\pi)^3$ is the lattice correction, 10^{23} the dimensional scaling. The mole is a lattice identity, not a convention.
P-AVG-6	The 2019 error is defined, not random. The kilogram was fixed via $h = 6.62607015 \times 10^{-34}$, which sits 812 ppm below $h_{FOT} = 5^3/(2 \cdot 3 \cdot \pi) \times 10^{-34} = 6.631455962 \times 10^{-34}$. N_A, derived from that kilogram, inherited the compounded offset and it was locked in, not corrected.
P-AVG-7	Pre-2019 measurements of N_A were converging toward the lattice value from above (1998 CODATA 6.022137×10^{23} = +536 ppm), consistent with the Radian Veil. The 2019 redefinition froze the value 537 ppm too high and terminated the convergence.
P-AVG-8	Measurement-then-fix inherits measurement error permanently; derivation-first eliminates it. The closure test (the Faraday ratio, and the closed loop) is a direct check requiring only arithmetic — it closes with N_A,FOT and not with N_A,SI.

A7. The Conversion Loop — the gears between the faces

from	to	operator	lattice form
energy (eV)	energy (kJ/mol)	÷ 10368	$2^7 \cdot 3^4$
energy (kJ/mol)	wavelength	÷ 36	$2^2 \cdot 3^2$
wavelength	free fall	÷ 49.50355350	$3888/25\pi$
free fall	frequency	× 6.283185307	2π
free fall	energy (J)	÷ 24	$2^3 \cdot 3$
wavelength	mass	× 1.233700550	$\pi^2/8$
energy (eV)	circumference	÷ 31104	$2^7 \cdot 3^5$
circumference	mass	÷ 22.00157933	$1728/25\pi$
free fall	speed of light	× 3110400 (=$g^2 \cdot$…)	$864 \cdot 3600$

These are the gears that turn one face of a T-value into another — a single quantity worn as energy, wavelength, free fall, frequency, mass or speed. The direct laws follow from them: $E = 6.822485557 \cdot m$; $\lambda = 0.810569469 \cdot m = 8m/\pi^2$; $eV = 373248 \cdot \lambda$ ($2^9 \cdot 3^6$); and mass→frequency $f = 0.102880658 \cdot m$ (25/243). Every step in the closed loop of Section 5 is one of these gears; with them a reader can reproduce any face-to-face crossing in this paper by hand.