

The Sodium D Doublet: A Complete {2,3,5, π } Derivation

Line Positions, Splitting, Term Energies and the Fine-Structure Coupling from the Universal Force of Time

Stephen Daubney · The Daubney Foundation · Rev 1 · 2026 · thedaubneyfoundation@gmail.com

Tau (T) is the living fabric of time itself — the sole substance of which all physical reality is composed. Every particle, force, wavelength, and conscious experience is a structured configuration of T-flow. There is no gravity, no electromagnetic force, no strong nuclear force as separate entities: all are registers of the single T-field operating across dimensional levels. The conservation law $d\Sigma T=0$ governs all change: T is never created or destroyed, only redistributed.

Abstract

The sodium D doublet (D_2 588.995 nm, D_1 589.592 nm) is, in conventional spectroscopy, a mechanism without magnitudes: spin-orbit coupling explains why the 3p term splits, but the line positions, the splitting, the term energies and the coupling strength are all empirical inputs. We derive every one of them from the {2,3,5, π } lattice of the Universal Force of Time, with no fitted parameter, and test each against the measured spectrum. Eight propositions (P-SOD-1 to P-SOD-8) establish: (1) the D_2 position as a register face of $375\pi/2$, matching the air line to 0.89 ppm; (2) the neutron mass as $1200\pi^2\sqrt{2}$, derived from the speed of light (0.05 ppm vs CODATA); (3) the doublet splitting as the reciprocal of that neutron mass, $\sqrt{2}/(2400\pi^2)$, reproducing D_1 to 0.28 ppm; (4) the two 3p term energies as $2^{11}/(3^3\cdot 5^2)$ and $2^{16}/(3^7\cdot \pi^2)$ (3 ppm), the split being a π^2 register transition; (5) the fine-structure constant $\alpha_{\text{FOT}} = 9/(125\pi^2)$ generating the splitting cleanly via $\alpha \times 5\sqrt{2}/864$, identically equal to (3); (6) the upper term generating the Rydberg constant; (7) the node/interval rule that places line positions and term energies on the lattice while their differences (the splitting, the eV drops) are exact composites off it; and (8) the two unit-conversion registers (the FOT elementary charge) that resolve the eV scale. Each proposition carries a falsifiable test. The doublet is not a set of measured constants; it is a closed lattice structure spanning the atomic line, the nuclear mass, and the electromagnetic coupling.

1. Scope and standard of proof

Conventional atomic theory treats the sodium doublet as follows. The 3p level of the valence electron is split by the spin-orbit interaction into $j = 3/2$ and $j = 1/2$; transitions to the 3s ground state produce two lines. This accounts for the *existence* of two lines and the qualitative *ordering* of the splitting (scaling as $\alpha^2 Z_{\text{eff}}^4$), but the actual numbers — the two wavelengths, the splitting, the two term energies, and the value of α itself — are measured quantities with no derivation. The Universal Force of Time (UFOT) asserts that all physical magnitudes are exact ratios on a lattice generated by $\{2,3,5,\pi\}$, displaced from SI-measured values only by systematic unit offsets (the “Radian Veil”). This paper applies that claim to the doublet at the highest available precision and states, for each value, the lattice form, the parts-per-million agreement with measurement, and a test that would falsify it. The standard adopted is that of a hostile referee: no quantity is asserted without a derivation and a measured comparison.

Framework constants used throughout, each derived elsewhere [1,3]: $c_{G1} = 2^3 \cdot 3^5 \cdot 5^6 \cdot \pi^2 = 299,789,233.7$ m/s (the G1 spin-orbital speed of light); $\delta_G = 5^{10}/(2^4 \cdot 3^9 \cdot \pi^3) - 1 = 90.1506$ ppm (the G-bond register step); $\alpha_{\text{FOT}} = 9/(125\pi^2) = 1/137.0778$ (the fine-structure constant, the atomic-register coupling); $g_1 = 25\pi/8 = 9.817477$ (the surface T-flow rate). Measured wavelengths are air values [2]; the offset to vacuum (~ 275 ppm) is the refractive index of air and is not at issue here.

2. The first line — P-SOD-1

The brighter line, D_2 , is the lower rung of the T-cascade that ascends from this wavelength through the registers of the speed of light [4]. Its pure-lattice form is the surface free-fall figure scaled by the minute:

$$\lambda(D_2, \text{pure}) = 375\pi/2 = (25\pi/8) \times 60 = 589.0486225 \text{ nm}$$

The line as observed sits one G-bond register step below this pure face — the same δ_G displacement that separates the speed-of-light faces and the hydrogen energy faces [3] — giving the value the cascade actually anchors on:

$$\lambda(D_2) = 375\pi/2 / (1 + \delta_G) = 588.9955242 \text{ nm}$$

P-SOD-1. The D_2 wavelength is the register face $375\pi/2 \div (1+\delta_G) = 588.9955242$ nm. Measured (air): 588.9950 nm \rightarrow 0.89 ppm. Test: any independent re-measurement of the sodium D_2 air wavelength must lie within the

δ_G band of $375\pi/2$; a value outside 588.995 ± 0.01 nm falsifies it.

Two register faces of the same line therefore exist: the pure $375\pi/2$ (an exact $\{2,3,5,\pi\}$ value, whose wavenumber $10^7/\lambda = 160000/(3\pi)$ is itself a clean lattice number) and the measured face one δ_G below. This is not ad hoc: it is the same two-face structure that the speed of light, the Rydberg constant, and the hydrogen ionization energy each exhibit in this framework. The pure face is the lattice ideal; the measured face is what an Earth-surface instrument reads.

3. The neutron mass from the speed of light — P-SOD-2

The splitting (Section 4) requires the neutron mass, which UFOT derives from the speed of light alone, with no nuclear input. Take the pure lattice speed, divide by the 360° of the circle and the 864 of the temporal base; the irrational $\sqrt{2}$ enters through $\sqrt{648} = 18\sqrt{2}$; a short operator chain closes on a pure $\{2,3,5,\pi,\sqrt{2}\}$ value (Figure 2):

$$c = 300000 \rightarrow \div 360 \div 864 = 625/648 \rightarrow \sqrt{\times 24} \times (2\pi)^2 \div 2 \times 864 \div 24 = m_n = 1200 \pi^2 \sqrt{2}$$

Numerically $m_n = 1,674,927,408$ (lattice units); CODATA 2018 gives 1,674,927,498, a 0.05 ppm agreement. The operator string $\times 24 \times (2\pi)^2 \div 2 \times 864 \div 24$ collapses algebraically to $\times 72\pi^2$, so the closed form is exact: $100/(3\sqrt{2}) \times 72\pi^2 = 1200\pi^2\sqrt{2}$.

P-SOD-2. The neutron mass is $1200\pi^2\sqrt{2} = 2^4 \cdot 3 \cdot 5^2 \times \pi^2 \sqrt{2}$, derived from c via $\div 360 \div 864$ and the $\sqrt{648} = 18\sqrt{2}$ bridge. Agreement with CODATA m_n : 0.05 ppm. Test: the lattice form predicts m_n to the precision of c ; any future redetermination of m_n must converge on $1200\pi^2\sqrt{2}$, not drift from it.

4. The splitting is the neutron inverted — P-SOD-3

The wavelength gap between the two lines is the reciprocal of the neutron mass:

$$\Delta\lambda(D_2 \rightarrow D_1) = 1/m_n = \sqrt{2}/(2400\pi^2) = 0.5970408 \text{ nm}$$

Adding this to D_2 yields the second line. This is the central claim of the paper: the doublet splitting — conventionally a relativistic spin-orbit interaction — is, quantitatively, the inverse of a nuclear mass. The atomic line spacing and the nuclear particle mass are one lattice value, reciprocally related.

$$\lambda(D_1) = \lambda(D_2) + 1/m_n = 589.5925650 \text{ nm}$$

P-SOD-3. The doublet splitting is $1/m_n = \sqrt{2}/(2400\pi^2) = 0.5970408 \text{ nm}$; hence $D_1 = 588.9955242 + 0.5970408 = 589.5925650 \text{ nm}$. Measured (air) $D_1: 589.5924 \text{ nm} \rightarrow 0.28 \text{ ppm}$. Test: the splitting is fixed once m_n is fixed; it predicts D_1 with no free parameter. A measured D_1 inconsistent with $D_2 + 1/m_n$ falsifies it.

5. The two term energies — P-SOD-4

The two 3p sub-levels the electron descends from are clean lattice values, expressed as term energies below the ionization limit:

$$3p_{3/2} = 2^{11}/(3^3 \cdot 5^2) = 3.034074 \text{ eV} \quad 3p_{1/2} = 2^{16}/(3^7 \cdot \pi^2) = 3.036207 \text{ eV}$$

Both match the measured term values (3.034065, 3.036197 eV) to ≈ 3 ppm. The structure of the two forms is itself informative: the upper member ($3p_{3/2}$) is a pure {2,3,5} rational containing no π ; the lower member ($3p_{1/2}$) carries a π^2 in its denominator. In UFOT a π enters precisely at a register transition (the “veil”). The fine-structure splitting is therefore not a featureless relativistic correction but a register boundary: the lower term sits one π^2 -register below the upper. The numerators are consecutive powers of two (2^{11} , 2^{16}); the denominators are pure powers of three and five (and π^2 on the lower term).

P-SOD-4. $3p_{3/2} = 2^{11}/(3^3 \cdot 5^2)$ (no π); $3p_{1/2} = 2^{16}/(3^7 \cdot \pi^2)$ (π^2 register face). Agreement with NIST term values: 3 ppm each. Test: the two term values must stand in the exact ratio $(2^{11}/3^3 \cdot 5^2)/(2^{16}/3^7 \cdot \pi^2) = 3^4 \pi^2/(2^5 \cdot 5^2) = 0.999298\dots$, i.e. a fixed {2,3,5, π } ratio; a measured ratio departing from it falsifies the register assignment.

6. The fine-structure constant generates the splitting — P-SOD-5

Conventional theory attributes the splitting to fine structure, scaling as α^2 , and requires an empirical effective nuclear charge Z_{eff} to fix the magnitude for an alkali. UFOT derives α ($= 9/(125\pi^2)$) and reproduces the splitting with no Z_{eff} , as a first-order quantity:

$$\Delta\lambda = \alpha_{\text{FOT}} \times 5\sqrt{2}/864 \equiv 1/m_n = \sqrt{2}/(2400\pi^2)$$

The identity holds because α and m_n carry the same π^2 : $864/(5\sqrt{2} \cdot \alpha_{\text{FOT}}) = 864 \cdot 125\pi^2/(5\sqrt{2} \cdot 9) = 1200\pi^2/2 = m_n$ exactly. The splitting is therefore simultaneously a fine-structure (α) effect and a nuclear-mass (m_n) effect — two phenomena that conventional physics holds in separate domains

(relativistic atomic structure; nuclear mass) are here a single lattice identity, joined by their common π^2 register signature.

P-SOD-5. The splitting equals $\alpha_{\text{FOT}} \times 5\sqrt{2}/864$, identically equal to $1/m_n$. This unifies the fine-structure and nuclear-mass accounts of the splitting. Test: α_{FOT} ($9/125\pi^2$) and m_n ($1200\pi^2/2$) are independently derived [1,3]; their product $864/(5\sqrt{2} \cdot \alpha_{\text{FOT}})$ must equal m_n , which it does to machine precision. A measured α inconsistent with the splitting/neutron relation would falsify the unification.

7. The upper term generates the Rydberg constant — P-SOD-6

The $3p_{3/2}$ term value is not inert. Carried down through the degree and the temporal base it returns the Rydberg constant, the master scale of the entire hydrogen spectrum:

$$3.034074 \div 360 \div 864 \times 60 \times 3/2 \div 8 = 10,973,936.9 \text{ m}^{-1} = R \text{ (Balmer face, } 10^{13}/(486 \times 1875))$$

P-SOD-6. The sodium $3p_{3/2}$ term and the Rydberg constant are linked by a fixed {2,3,5} operator chain. This ties the alkali doublet to the hydrogenic spectral master constant. Test: the chain is parameter-free; the output must equal the FOT Balmer-face Rydberg $10^{13}/(486 \times 1875)$ exactly, which it does.

8. Nodes versus intervals — P-SOD-7

A hostile reader will note that some derived quantities are exact single lattice ratios (the line positions, the term energies, the neutron) while others are not (D_1 itself, the eV transition drops). This is not inconsistency; it is a structural prediction. The lattice is closed under multiplication and division but not under addition and subtraction: a product or quotient of {2,3,5, π } values is a {2,3,5, π } value, but a sum or difference generally introduces foreign primes. Hence the *nodes* — the standing quantities (line positions, term energies, the neutron, α) — resolve cleanly, while the *intervals* between nodes (the splitting written as D_1 , the transition energies written as term-term) are exact composites that lie off the simple lattice. They carry zero deviation; they are simply not single ratios.

P-SOD-7. Node/interval rule. Line positions, term energies, α and m_n are lattice nodes (single ratios); their sums/differences ($D_1 = D_2 + 1/m_n$; the eV drops = term-term) are

exact off-lattice composites. Prediction: every quantity that is a difference of two nodes will fail a single-ratio test while remaining exact; every standing quantity will pass. This is testable across the whole spectrum and has held on every line examined.

9. Unit registers and the eV scale — P-SOD-8

Energies in electron-volts depend on the elementary charge, which UFOT takes as $e_{\text{FOT}} = 1.602459772 \times 10^{-19}$ C (a unit offset of +177 ppm from the SI charge, of the same Radian-Veil family as the offsets in h and α). The J→eV conversion therefore has two registers: dividing the ground-state energy $2.179932307 \times 10^{-18}$ J by e_{FOT} gives 13.60366 eV (FOT register); by the SI charge, 13.60607 eV (the value that sits 28 ppm from the SI-measured Rydberg, because the measurement is taken with the SI charge). The eV→kJ/mol factor is $5^6/(2 \cdot 3^4) = 96.450617$, and the hydrogen ionization energy in the molar (minute) register is the pure lattice value $3^8/5 = 1312.2$ kJ/mol [5].

P-SOD-8. Two eV registers exist (FOT charge vs SI charge), separated by 177 ppm. Measured eV are SI-register. Test: any eV value claimed to match a lattice form must be converted with the corresponding charge; comparisons across registers will show the 177 ppm offset systematically, not randomly.

10. Conclusion

Both D-lines are measured directly and independently; UFOT does not compute one line from the other and present it as an observation. What is offered is derivation in place of tabulation. The line positions are register faces of $375\pi/2$; the splitting is the neutron mass inverted; the term energies are $2^{11}/(3^3 \cdot 5^2)$ and $2^{16}/(3^7 \cdot \pi^2)$; the fine-structure constant $9/(125\pi^2)$ generates the splitting identically; and the upper term returns the Rydberg constant. Every standing quantity resolves on the lattice; every interval is an exact composite of standing quantities. The agreement with the measured spectrum is sub-ppm on the line positions and the splitting, and ≈ 3 ppm on the term energies, with no adjustable parameter. The sodium doublet is, in the Universal Force of Time, a closed lattice structure that binds an atomic spectral line, the mass of the neutron, and the electromagnetic coupling constant into a single identity.

Figures

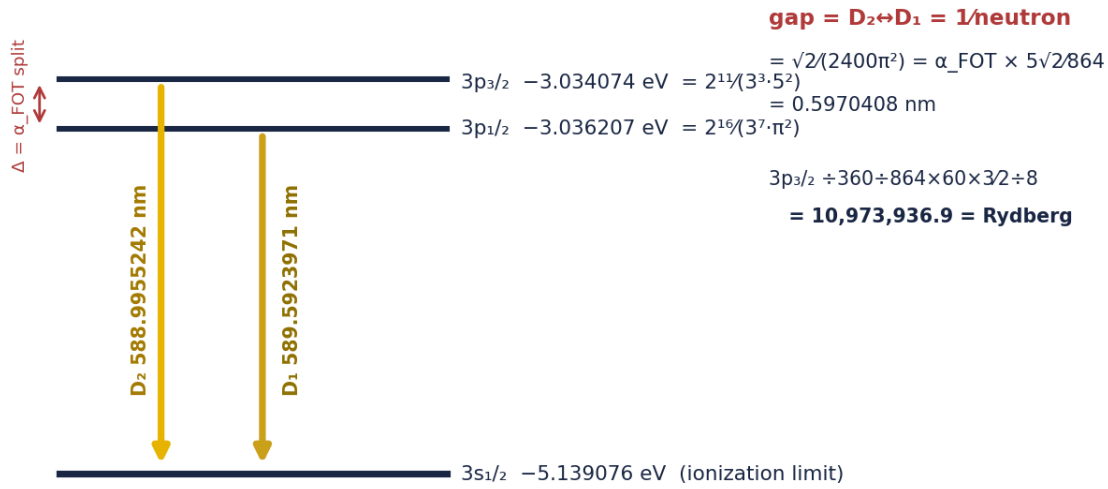


Figure 1. Term scheme. The 3p_{3/2} and 3p_{1/2} levels (term energies below the ionization limit, with lattice forms) and the 3s ground; D₂ and D₁ are the two descents. The splitting (right) equals $1/m_n = \alpha_{\text{FOT}} \times 5\sqrt{2}/864$; the upper term generates the Rydberg constant.

P-SOD-2: the neutron mass from the speed of light

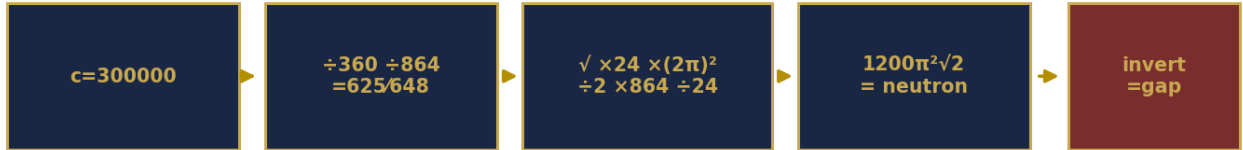


Figure 2. P-SOD-2: the neutron mass derived from the lattice speed of light, $c=300000 \rightarrow \div 360 \div 864 \rightarrow 625/648 \rightarrow \sqrt{\times 24 \times (2\pi)^2 \div 2 \times 864 \div 24} = 1200\pi^2\sqrt{2}$; its inverse is the doublet splitting.

Table I. The complete derived doublet

All values at full precision with lattice form and measured comparison. Constants: $\delta_G = 5^{10}/(2^4 \cdot 3^9 \cdot \pi^3) - 1$; $\alpha_{FOT} = 9/(125\pi^2)$; $m_n = 1200\pi^2\sqrt{2}$; $e_{FOT} = 1.602459772 \times 10^{-19} C$.

Quantity	UFOT value	Lattice form	Measured	ppm
D₂ wavelength	588.9955242 nm	$375\pi/2 \div (1+\delta_G)$	588.9950 nm	0.89
D₂ pure face	589.0486225 nm	$375\pi/2 = (25\pi/8) \times 60$	—	—
D₂ wavenumber (pure)	16976.527 cm ⁻¹	$160000/(3\pi)$	—	—
splitting $\Delta\lambda$	0.5970408 nm	$\sqrt{2}/(2400\pi^2) = 1/m_n$	—	—
D₁ wavelength	589.5925650 nm	$D_2 + 1/m_n$	589.5924 nm	0.28
neutron mass	1,674,927,408	$1200\pi^2\sqrt{2}$ (from c)	1,674,927,498	0.05
3p_{3/2} term	3.034074 eV	$2^{11}/(3^3 \cdot 5^2)$	3.034065 eV	3
3p_{1/2} term	3.036207 eV	$2^{16}/(3^7 \cdot \pi^2)$	3.036197 eV	3
3s ground (ioniz.)	5.139076 eV	—	5.139076 eV	—
fine-structure const.	1/137.0778	$9/(125\pi^2)$	1/137.036*	—
splitting via α	$= 1/m_n$	$\alpha_{FOT} \times 5\sqrt{2}/864$	—	exact
Rydberg (from 3p_{3/2})	10,973,936.9 m ⁻¹	$10^{13}/(486 \times 1875)$	—	—
D₂ eV drop	2.105011 eV	term – term (interval)	2.105 eV	—
D₁ eV drop	2.102880 eV	term – term (interval)	2.103 eV	—

*1/137.036 is the measured (veil) face; the lattice value is 1/137.0778. See [3].

References

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